Forty years without Cavendish

Francesco Castaldi - Unione Astrofili Italiani - francesco.castaldi1@gmail.com

Abstract: This study deals with several misunderstandings about the development of the attraction law between two masses. Every student learns that Newton wrote an equation with the product of those masses and reciprocal to the square of their distance, all multiplied by a certain constant G. And, as a conclusion, the numerical value of this G should have been discovered by Cavendish through a celebrated experiment.
As a matter of fact, Newton presented that force of attraction as due to the bulk of those masses, but exactly proportional to the inverse square ratio of distance between the centres of these two masses. We're aware that Cavendish, on 1798, published the mean density value of the Earth compared to the unit density of water. He never spoke about the product of masses and that constant G.
At those times, great mathematicians like Lagrange and Laplace, knowing with good precision the Earth volume, could calculate the mass of the Earth. But Cavendish's data was not approved by his contemporaries and after, for a total of forty years, with the exception of Laplace. When the experiments on gravity attraction were repeated, in similar or different conditions, Cavendish's value returned as the best and was finally approved by everyone. Regarding the so-called Newton's equation, it appears as a mathematical opportunity; that is, to write a simple expression for the gravity attraction between two masses by means of their product. Consequently, around 1870, a dimensional role emerged played by a universal constant, by us known as G.
Here one thing is put in evidence, that is that, before this final agreement, Newton's equation had been expressed by S.D. Poisson in 1811, signifying its mathematical necessity, but only later it became the numerical formula we know to quantify the force of gravity.

Keywords: Henry, Cavendish, density

1. Introduction

This subject-matter starts by *Principia*, Book III, Proposition X, with these words (Newton 1803, p. 181): "it's probable that the quantity of the whole matter of the earth may be five or six times greater than if it all of water; especially since I have before shewed that the earth is about four times more dense than Jupiter."[1]

[1] Modern data: Earth Δ=5.515; Jupiter Δ=1.33, that is Earth density 4.15 times Jupiter's.

We know the methods used in order to verify the mean density Δ of the Earth, but there was a strange "anachronism", because now we're used to read something of this tenor:

> The law of universal gravitation states that, when the masses are spheres with centres d apart, this attraction is GMM'/d^2, G being a constant – the gravitation constant – the same for all masses. […] two sphere of known mass and dimensions, as in all the various forms of Cavendish's experiment. Knowing the gravitation constant G, we may at once find the mean density of the Earth Δ. (Poynting 1891, p. 566)

On the contrary, experimenters of 18^{th} and 19^{th} centuries went on looking for a numerical value of this mean density Δ, while only towards the end of the 19^{th} century they considered the need to get a value for G.

2. Learned people's scepticism

In the outskirts of mount Schehallien, Scotland, several researchers made experiences with the aid of a plumb line. In 1774 N. Masqueline produced a series of measurements regarding gravitational attraction (Masqueline 1775, pp. 500-542); thanks to them, C. Hutton calculated Δ=4.5. (Hutton 1778, pp. 689-788) On a second time, in a 'Letter to Laplace', he claimed to be the author of inherent calculations and decided Δ=5.04. (Hutton February 1820)

But Cavendish had been insuperable publishing in 1798 a relation of his experiments by means of a torsion balance built, on behalf of the Royal Society, after a model of John Michell. (Cavendish 1798, pp. 469-526)

At the end, Cavendish arrives at a conclusion: "the density of the earth comes out 5,48 times greater than that of water." (Cavendish 1798, pp. 520-522) The problem arose with learned people reluctant to accept this data and unable to choose who-did-what:

> By means of this experiment, Cavendish made the law of universal gravitation full. This law was no more a proportionality relation, as Newton enunciated it, but a precise law able to make possible a quantitative analysis. It was the most important contribution to gravitational study since Newton's times. (Frautschi *et al*. 1992, p. 136)

Also the Rizzoli-Larousse encyclopaedic dictionary at the headword 'Cavendish' writes a similar opinion: "In 1798, by means of a torsion balance, named after him, he measured the universal gravity constant deducing from it the mean density of the Earth." (Rizzoli-Larousse 1967, Vol. III)

On the contrary, Cavendish's contemporaries, and for several decennials after his death in 1810, barely accorded him an appreciation on the subject.

3. In the first half of XVIII century something is moving

Almost all the planets mass were written according to a unitary Sun's mass. Newton opened this road (*Principia*, Book III, Proposition VIII, Theorem VIII); his data were improved (Laplace 1802, p. 61) and we find them once more in 1818 (Laplace 1818, p. 48), so that in the last edition, (Laplace 1824, p. 239), these values have an increased precision, but are always referred to the unitary Sun's mass.

John Herschel (Herschel 1833, p. 416) copied the same data, referred to different unit measurements.

Never Laplace was closed into his results and wrote (Laplace 1808, Tome II, Livre IV, Chap. VIII, p. 130) that the Earth is a sphere having 6369.374 km of radius, so that the volume is V=1081638 km^3; as well (Laplace 1808, p. 147) that Cavendish found for the Earth, '*à fort peu près*', Δ=11/2 the water density in kg/dm^3 and here he stopped, without drawing $M=V*\Delta=5.95*10^{21}$ kg. Actually, with the modern Earth's data, V=1.083.320 km^3 and density 5.515 kg/dm^3, we get the mass $5.976*10^{21}$ kg. Everybody should have celebrated Laplace's work, giving the mass of every component of solar system through the Sun's mass. Not even the posthumous edition of 1841 contains such information, and Laplace repeats (p. 303) the same descriptions about Cavendish's work.

Previously, in an article we found (Laplace 1820, pp. 328-331): "Through a scrupulous examination of Cavendish's apparatus and all his tests made thanks to the precision and to the skill distinguishing this excellent physicist, I don't see any critics to do against his data which gives 5.48 for Earth's density […]". Apart from Poisson of whom we'll talk about later, we haven't found a former appreciation of this data, also if he didn't decide to render the real value of masses.

That notwithstanding, Laplace's name woke up the physicists awareness (Airy 1834, p. 2): "The most remarkable experiments which prove that bodies attract each other are a set of experiments made at the end of last century by Mr. Cavendish." And after a short description about the equipment, he goes on: "But the results of this experiment are striking, principally because they are unusual; the ordinary force of gravity serves quite as well to prove the existence of some such power." But he doesn't mention Cavendish's data, and this is a residual distrust over this '*unusual*' result.

Finally, in 1842, A. Cournot translated into French, with modifications and additions to his first edition of 1834, *A Treatise on mechanics*, of Kater & Lardner (Kater, Lardner 1831), members of the Royal Society of London.

It seems strange that in France, for "teaching elementary knowledge of mechanics in Normal Schools of first degree and other organizations for teaching", there was nothing better than a translation from an American book; however Cournot illustrates the experiment and concludes (Cournot 1842, p. 109): "Cavendish found the average density of the Earth about 5.5 times that of the water." Then he writes (Cournot, p. 112): "The Earth radius, or the distance of terrestrial surface from its centre is about 637 miriameters […]" our 6370 km, an average value between polar and equatorial radiuses, but the Earth mass is not deduced. Here and in other parts, Cournot attended over the text in order to give it a greater precision.

It's interesting to return to the original test where Cavendish's experiment is summarized (Kater, Lardner, p. 69), but with no value of Δ, because for them this was a question 'regarding the Physical Astronomy'. Incidentally, in 1851 also an Italian translation from Cournot was printed.

4. The return to the torsion balance

On Table 1 the experiments realized by means of torsion balances, plus one in coal mines.

Authors	Pubblications	Density	Year
1-Cavendish	*Philosophical Transactions*, 1798, Part. II, p. 469-526	5.48	1798
2-Reich F.	*Philos.Magazine,* vol. XII, gen-giu 1838, p. 284 (Baily)	5.44	1837
3- Baily F.	*Philosophical Magazine*, Vol. XXI, 1842, p. 111	5.67	1842
4-Reich F.	*Philos. Magaz.*, Vol. V, 4^a Serie, Jan-Jun 1853, p. 157	5.58 ∓ 0.015	1853
5-Airy G.B.	*Pendulum Experiments*, *Philos.Transact.*, *Part 1*, p. 46	6.56 ∓ 6 0.182	1856
Actual data		5.515	2000

Table 1. Experiments for Earth's average density determination (water =1)

We are obviously surprised by the fact that 40 years passed and no other experiments were done since 1798. And consequently we want to know why it happened and which consequences produced the return to the tests.

1. During a meeting of the Royal Astronomical Society [R.A.S.] in Nov. 10th 1837, vice president F. Baily, made a report of news arrived through European magazines over experiments conducted by Physics professor F. Reich in Saxony. (Baily 1838, p. 283-284) At first, the meeting of R.A.S. had been organized "on the repetition of the Cavendish Experiment, for determining the Mean Density of the Earth", so that "a Commettee was appointed more than two years ago to consider of its practicability"; finally the R.A.S. got a result "Her Majesty's government having been pleased to grant the sum of 500 *l.* towards defraying the requisite expenses." As a conclusion of this meeting, Baily writes that R.A.S. works shall go on.
   But Reich had announced his success in a memoir on September 1837 during a meeting of German Scientific Association in Prague: its apparatus contained only one sphere, at first of lead, then of cast iron, 45 kg of weight. After two years of preparations, 57 tests at the torsion rod, developed in the months of June, July and August 1837, he gave an average density at first 5.44, then 5.43.

A year later, (Reich 1838) he printed a report in a booklet: ‘*Experiments on the average Earth density by means of a torsion balance*’.

2. Only four years later Baily reports to the R.A.S. about his experiments, 2004 in total, of 13th of May and 10th of June, 1842, (Baily 1842, pp. 111-121), thanks to which the mean density of the Earth was established Δ=5.67. The columnist takes some precaution remembering that Cavendish intended to repeat his experiments, never executed, so that those of Baily were a kind of continuation.[2]

3. In 1853 the same Reich refers about three series of new tests by different torsion cables. (Reich 1853, pp. 153-159) At the beginning of his article Reich seems to mock: “I ought perhaps to apologize for returning to a subject which has been already submitted to so fundamental an examination, that it may seem superfluous to enter upon its further consideration.” Starting from the persisting behaviour of learned physics the situation appears now the opposite.

4. The tests in English coal mines of G.B. Airy deserve a particular chapter: his project was to install two simple pendulums inside bases at difference in height of 360 m. But he abandoned the project after two failed tests:
   - a fire in the box containing the pendulum to lower down, 1826;
   - the flooding of the inferior station where the pendulum was yet installed, 1828.

Only looking at the difficulties faced by Baily in the repetition of Cavendish’s tests, Airy decides in 1854 to return to his project and then to print his results. But he gets a density Δ=6.656±0.182, too far from the one he obtained in the past with the torsion tests and he is forced to look for a justification:

> 64. The value thus obtained is much larger than that obtained from the Schehallien experiment, and considerably larger than the mean one found by Baily from the torsion rod experiments. It is extremely difficult to assign with precision the causes or the measures of the errors of any of this determination; and I shall content myself with expressing my opinion, that the value now presented is entitled to compete as the others, or, at least, equal terms. (Airy 1856, p. 46)

Among scholars distrust towards Cavendish’s data is evident.

Incidentally, in 1871 there is an announce made by father A. Secchi to produce verifications on the Alps by means of the tunnel of Frejus under construction, making use of two stations with a difference in height of 1600 m; but the developments of this projects are not known. (Radau 1880, p. 19, in a footnote)

[2] At p. 113 we read that, for his magnetism studies, Gauss introduced the little mirror in middle position of the torsion cable, then copied by Reich; it was not a Cavendish’s device, as illustrated in several modern school books.

5. The problem's clarity in the second part of the eighteenth century

In 1858, the Earth's dimensions had no secret for superior schools students. (Delaunay 1855, p. 203)

For the moment the Planets' masses are still written in fraction of the unitary value of the Sun (p. 549) which are not much different from the Laplace one, but finally the progress enters in the history with the name of Cavendish, without any reference to the scandal of the delay:

> §315. Mean density of Earth – The universal gravitation's theory gave the possibility to find the masses of Sun and planets in relation to one of them taken as unity (§298). Here from, it's sufficient the mass of one of this bodies, related to the masses we see around, so that a complete knowledge of all other masses be consequent. Obviously it is over the Earth that this determination must guide: instead of looking for a number representing its total mass, it's better to look for the determination of the mean density of this globe […]; actually, it'll be enough the combination of the mean density of the Earth with his volume to arrive to its mass value. The mean density was determined by Cavendish. (Delaunay 1855, p. 585)

Here an illustration of the famous experiment is printed with exactly the same original drawings printed in *Philosophical Transactions*, 1798. The subsequent paragraph considers the mean density of the Sun Δ=1.37, slightly (0.04) less of the actual value: "§316. On the planets density – The mean density of the Earth gives the possibility to find similarly the mean densities of Sun, Moon and planets." Referring to a previous table of the Sun with unitary density [p. 549], Delaunay makes a second Table with densities which are not far from modern data, with the exceptions of Mercury and Mars. Inside the text we neither find a reference to the universal constant G, nor to the so-called Newton's equation, evident indication that, at the end, it was the Δ, consequent to the experiments of Cavendish, to persuade about the so-called Newton's equation containing the constant G.

One year later, 1859, we have a similar example with tables (Secchi 1859, pp. 40, 42): the planets mass are still related to the Sun's one, with some amelioration compared to Laplace; there are diameters and density of everyone, in particular for the Earth 5.55 and for the Sun 1.42.

The breach is opened and another expert, after descriptions of experiments and a great praise of Cavendish's work, writes (Hoefer 1873, p. 515): "We know the radius and the mean density of terrestrial Globe, so that it's easy to learn the weight of the total mass of our planet. The Earth's mass is nothing else than 354936th part of the Sun one [the same Laplace's value after 50 years]; in other words it would be necessary a weight of 350000 times that one of the Earth in order to balance, on the scales pan, the Sun mass." And here he stops his speech.

In the year 1881 this value was, in a good approximation, the modern one. Anyway, it's impossible not to think that Laplace was ready, at the beginning of 19th century, to make clear these data for the Sun and consequently for all planets.

6. At the end, Δ and G together

In the meantime, the experiments due to the torsion balance went on and the results were expressed for both Earth mean density and gravity constant, Table 2.

Author	Publication	Dens. Δ	G	year
Cornu&Baille	*Comptes Rendues*, Tome 66, 1873, p. 957	5.50	6.578	1873
Poynting J.H.	*Philosoph. Trans.*, vol. 182, 1891, pp. 565-566	5.4934	6.6984	1891
Boys C.V.	*Philosophical Trans.,* vol. 186, 1896, p. 2	5.527	6.657	1895
Current data		5.515	6.67	2000

Table 2. The last experiments to determine the mean Earth density Δ and the universal constant G (in MKS $*10^{-11}$)

Finally some relations become of public domain (Fig. 1).

> **(*) En effet, si l'on applique la loi de Newton à deux corps quelconques, de masses m, m', on a $F = fmm' : r^2$; si l'un d'eux n'est autre que la Terre, on a $p' = fMp' : gr^2$ ou $f = gR^2 : M$.**

Fig. 1. In this note (Cornu, Baille 1873, p. 954) has a detailed well known opinion[3]

Substituting *f* where we normally write G as the gravitation constant, we return to what had been written by S.D. Poisson (1811, Tome II, p. 16, p.34) at the end of a long series of verbal arguments: "321. The motive power of mass M due to the attraction of the mass m, is represented by $\frac{Mm}{R^2}f$." Here we read the so called Newton's equation written for the first time. It is represented by le following parameters:

- *M*, Earth mass, a product of the terrestrial density Δ and of the volume $V = \frac{4}{3}R^3$
- *m*, an any mass on Earth surface so that it is far *R*, terrestrial radius, from its centre;
- By means of the 2nd law of Newton, as expressed by L. Euler, 1736, Poisson could obtain *f*;
- $f\frac{Mm}{R^2} = mg$; from here, the unknown value *f* gets an actual (and numerical) value: $f = \frac{R^2 g}{M}$.

The choice of the product '*M*, *n*' reveals to be a 'mathematical opportunity'. It's the motive force cited by Poisson who stopped himself at this very point, also if he knew very well the 'beautiful experiment' of Cavendish, giving the bibliographic reference and a praise of the numeric result.

---

[3] Translation: Actually, if someone apply the Newton's law to two whatsoever bodies, of m and m' masses, he gets $F = fmm'/r^2$; if one of these masses is simply the Earth, he gets … $f = gR^2/M$.

Poisson doesn't write which kind of practical meaning was to give to *f*, unless he had the same opinion of a modern historiographer (Gonzales 2001, p. 531): 'a dimensional role'. The Earth's mass $M = \Delta\frac{4}{3}R^3$ is given by known parameters, so that it is difficult to justify the phrase described by Cornu and Baille in Fig. 2: "The mass absolute value of celestial bodies, necessary to know their density, is not possible unless through the determination of the absolute mass or of the mean density Δ of the Earth bound to the attraction constant by means of the formula."

La valeur absolue des masses des corps célestes, nécessaire pour la connaissance de leur densité, n'est possible que par la détermination de la masse absolue ou de la densité moyenne Δ de la Terre liée à la constante de l'attraction par la formule (*)

$$f\Delta = \frac{3}{4}\frac{g^2}{\pi R}.$$

» C'est donc à la détermination de *f* que se réduit, en dernière analyse, la solution de cet important problème, et la méthode expérimentale con-

Fig. 2. Citazione da Cornu, Baille (1873)

The phrase written under the formula could be taken as a knowledge display: "Therefore, it's in the determination of *f* that the solution of this important problem, at the end, is reduced […]."

Cornu, Baille (1873, p. 957) summarize the data of their activity with the torsion pendulum in order to obtain f/g^2 and consequently Δ, while the historic development of science didn't happen in this manner.

7. Conclusion

There aren't doubts in the determination of the experimental data Δ of Cavendish as supporter of G, after its introduction in an "appropriate" equation due to Poisson, 1811.

As to the so-called Newton's equation, from his words it is reasonable to write: F=constant/r^2. About this attractive force in relation to masses Newton gives several examples.

Proposition 6, Theorem 6 I. (Newton 1729, Vol. II, Book III, p. 220): "That all bodies [on the surface] gravitate towards [the centre of] every planet; and that the weights of bodies towards any the same planet, at equal distances from the centre of the planet, are proportional to the quantities of matter which they severally contain." And so on for other propositions.

Other interesting Proposition 75, Corollary 1 (Newton 1803, Vol. I, Book I, Section XII, p. 179): "The attractions of sphere towards other homogeneous spheres are as the attracting spheres applied to the squares of the distance of their centres from the centres of those which they attract." While the Corollary 2 ends: "and therefore since in all attractions (by law 3) the attracted and attracting point are both equally acted on, the force will be doubled, by their mutual attraction, the proportions remaining." "It will be doubled" is the literal translation of *geminabitur*; but with the meaning of the same mutual attraction repeated.

The reference to 'law 3' is simply the 3rd Law of Motion, exclusively Newtonian (Newton 1803, Vol. I, Book I, p.15): "To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal and directed to contrary parts." The force by which the Sun attracts the Earth produce an equal one by which the Earth attracts the Sun; so that the absolute value of this force, is the unique force in discussion when there are two bodies interactive by means of their masses.

He clearly rendered his own opinion in the 2nd edition, 1713, of *Principia*, in a *General Scholium*:

> This is certain, that it must proceed from a cause that penetrates to the very centres of the Sun and Planets, without suffering the least diminution of its force; that operates […] according to the quantity of the solid matter which they contain, and propagates its virtue on all sides, to immense distances, decreasing always in the duplicate proportion of the distances. (Newton (1729, Vol. II, Book III, p. 392)

Soon after in the same page of *Principia*, Newton ends, and we with him:

> But hitherto I have not been able to discover the cause of those properties of gravity from phenomena and I frame no hypotheses [*hypotheses non fingo*]. […] And to us it is enough , that gravity does really exists, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and for our sea.

References

Airy G.B. (1834). *Gravitation: an explanation of principal perturbations in the solar system*. London: Knight.

Airy G.B. (1856). "Account of Pendulum experiments undertaken in the Harton Colliery". *Philosophical Transactions of the Royal Society of London, Part. I*, 146, pp. 297-355.

Baily F. (1838). "On the Repetition of the Cavendish Experiment, for determining the mean density of the Earth". *Philosophical Magazine*, XII, pp. 283-284.

Baily F. (1842). "An account of some experiments with the torsion-rod for determining the mean density of the Earth". *Philosophical Magazine*, XXI, pp. 11-121.

Castaldi F. (2006). "Fra leggi fisiche ed equazioni matematiche". *Astronomia*, 2, pp. 4-10. Proceedings of the XXXVIII Congress, of Unione Astrofili Italiani UAI, La Spezia, 2005.

Cavendish H. (1798). "On the Method of computing the Density of the Earth". *Philosophical Transactions of the Royal Society of London*, 69, pp. 469-526.

Cornu A., Baille J. (1873). "Determination nouvelle de la constant de l'attraction et de la densité moyenne de la Terre". *Comptes Rendus*, LXXVI, pp. 954-958.

Cournot A. (1842). *Eléments de mecanique*. Paris: Mathias.

Delaunay C. (1855). *Cours elementare d'astronomie*, Paris: Langlois & Leclercq.

Frautschi S.C., Olenick R.P., Apostol T.M., Goodstein D.L. (1988). *L'Universo meccanico*. Bologna: Zanichelli.

Gonzales A.M. (2001). "Weighting the Earth: a Newtonian test and the Origin of an Anachronism". *Science and Education*, 10, pp. 515 543.

Herschel J.F.W. (1833). *A treatise on Astronomy*. London: Longman.

Hoefer F. (1873). *Histoire de l'Astronomie.* Paris: Hachette.

Hutton C. (1778). "Calculations to ascertains the density of the Earth". *Philosophical Transactions of the Royal Society of London*, 68, pp. 689-788.

Hutton C. (1820). "A letter of Dr. Hutton to the Marquis de Laplace on several astronomical and philosophical subjects". *Philosophical Magazine*, 55, pp. 81-85.

Kater H., Lardner D. (1831). *A Treatise on mechanics*. Cambridge: Hilliard and Brown.

Le Seur T., Jacquier F. (1760) (eds.). *Philosophiae Naturalis Principia Mathematica Auctore Isaaco Newtono.* Coloniae Allobrogum: Philibert.

Laplace P.S. (1802). *Traité de Mécanique Celeste, Livre VI, Tome III*. Paris: Duprat.

Laplace P.S. (1808). *Exposition du système du monde, 3rd edition*. Paris: Courcier.

Laplace P.S. (1820). *Sur la densité moyenne de la terre* in *Connaissance des Tems ou des mouvements célestes pour l'an 1823*. Paris: Courcier, pp. 215-220.

Laplace P.S. (1818). *Exposition du système du Monde, Tome II.* Paris: Imprimerie Royale.

Maltese G. (1992). *La storia di "F=Ma".* Firenze: Olschky.

Maskelyne N. (1775). "An account of observations made on the mountain Schehallien for findng its attraction". *Philosophical Transactions of the Royal Society of London*, 65, pp. 500-542.

Newton I. (1803). *The Mathematical Principles of Natural Philosophy, Translation of Motte A.* London: Symonds.

Poisson S.D. (1811). *Traité de Mécanique, Tome II*. Paris: Courcier.

Poynting J.H. (1891). "On determination of mean density of the Earth and the gravitation". *Philosophical Transactions of the Royal Society of London*, 182, pp. 565-656.

Radau M.R. (1880). *La constitution interieure de la Terre.* Paris: Gauthier-Villars.

Reich F. (1838). *Versuche über die mittlere Dichtingkeit der Erde mittels der Drehwage*. Freiberg: Engelhardt.

Reich F. (1853). "New experiments on the mean density of the Earth". *Philosophical Magazine*, 5 (31), pp.153-159.

Secchi A. (1859). *Quadro fisico del sistema solare*. Roma: Tipografia Belle Arti.