The three formulations of quantum mechanics founded on the alternative choices

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Abstract: A previous paper obtained a classification of around dozen formulations of quantum mechanics according to the four choices on the two dichotomies constituting the foundations of science. The great majority of them correspond to the dominant choices, i.e. to the Newtonian choices. Only three formulations are recognised as linked to the alternative choices: Heisenberg-Born-Jordan's one, F. Jordan's recent one and Bub's one. These formulations are examined. No one is recognised as consistently formulated according to the alternative choices. As a conclusion, the formulation of quantum mechanics that is alternative to the dominant ones has still to be invented. An appendix gives reason of the general use of both analogies and the correspondence principle during the first period of the history of quanta; they constituted the sole resource for reasoning. Actually they belong to intuitionist logic.

Keywords: Quantum mechanics, formulations, fundamental choices, alternative choices, Heisenberg, Born, Jordan, F. Jordan, Bub, analogies, correspondence principle, intuitionist logic

1. Introduction

In the literature around dozen formulations of quantum mechanics (QM) are known, their number being different according to what one means for "formulation". Previous my papers have suggested that the foundations of a scientific theory are constituted by two dichotomies; one on the kind of infinity, either the actual one (AI) or the potential one (PI); another on the kind of a theory organization, either one deducing logical consequences from certain axioms (AO), or one looking for a new method capable to solve a crucial problem (PO), or, equivalently, either an organization using the classical logic or the intuitionist logic (Drago 1990).

A detailed and long analysis is usually required for arriving to decide which are the basic choices of a physical theory (recall how difficult was to decide the basic notions and choices of Newtonian mechanics). However, already some scholars have suggested categories capable to class the variety of QM formulations. In a previous paper (Drago 2014), these categories have been compared to the two basic dichotomies. This comparison allows to roughly characterize the two basic choices of each QM formulation. The result is represented by the following wind rose graph (where each of the names –

Lagrangian, Newtonian, Carnotian and Descartesian – characterizes a pair of choices on which of the more celebrated theory by each of these physicists was founded).

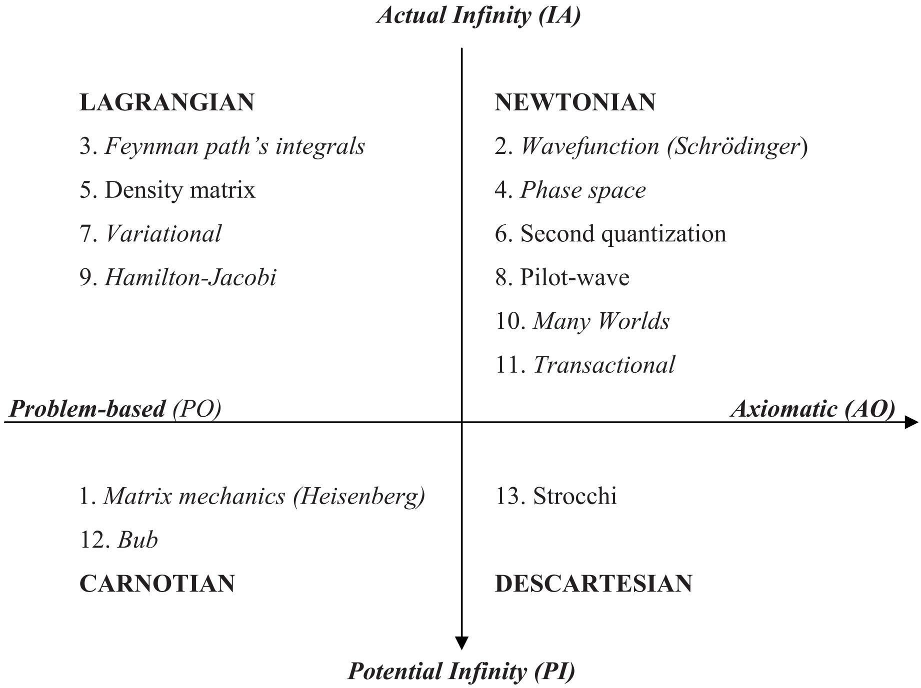

Fig. 1. Different formulations of Quantum Mechanics. In Italics the formulations to which the attribution of the two choices is more certain

Let us remark that, out of the represented 13 formulations, the great majority of them (10) belong to the two higher quadrants. This means that almost all the formulations choose an idealistic mathematics (AI); their authors seem to ignore any alternative in mathematics. Rather, they choose between the Newtonian model and the Lagrangian model – actually between AO and PO – only for technical reason of higher theoretical efficiency.

The majority of these 10 formulations (6) belong to the Newtonian model. This fact means that the revolutionary step of the 20^{th} Century – to have overcome Newton's mechanics – did not led the QM theorists to abandon its model; in particular, the usual formulation 2 of QM agrees with the Newtonian model.

Only one formulation belongs to the lower right quadrant, and three formulations to the other lower quadrant. In sum, this distribution shows that the dominant formulations of QM represent a backwards theoretical attitude, and moreover that they as a whole insufficiently represent the essential pluralism of theoretical physics.

In the lower left quadrant are located three formulations sharing, *grosso modo*, the same two fundamental choices, which are the alternative choices to the dominant ones: the constructive mathematics, instead of the classical one; and the problem-based organization, instead of the deductive one; or, equivalently, the use of the intuitionist logic instead of the classical logic. In the following these formulations will be analysed in order to verify how much their theoretical developments are consistent with their two basic choices.

An appendix gives reason of the use of mainly analogies and the correspondence principle during the first period of the history of quanta; they were the sole resource for reasoning; actually they belong to intuitionist logic.

2. Heisenberg's Matrix Mechanics

In the history of QM Heisenberg-Born-Jordan's Matrix Mechanics, (van der Waerden 1968, pp. 261-415) was the first formulation. The original idea was suggested by Heisenberg, and then developed by Born and Jordan by means of the matrix formalism.

The basic assumption of Heisenberg's formulation is to rely on the observable magnitudes only, hence to conform the theory to an operative attitude; in philosophical terms, this requirement equates the constructivity of the mathematics. In fact, the subsequent Born and Jordan's choice was the mathematics of discrete matrices, i.e. mathematical objects based on PI, and presumably manipulated in agreement with constructive mathematics. However an undecidable problem may arise in the case of multiple solutions of the eigenvalue problem of a matrix equation, unless more properties allow splitting them (Aberth 1971). An analysis has to be performed about the occurrence of this problem in matrix mechanics.

Surely, the kind of the organization is a PO for several reasons. The formulation is developed in order to solve a problem, i.e. the great problem of describing quantum reality. As a fact, the principle of indeterminacy – of course a limitation principle – plays a basic role. Heisenberg's paper lucidly states a different, but equivalent problem:

> The question therefore arises whether, through a more precise analysis of these kinematic and mechanical concepts, it may be possible to clear up the contradictions evident up to now in the physical interpretations of quantum mechanics and to arrive at a physical understanding of the quantum-mechanical formula (Heisenberg 1927, p. 63).

He made use of doubly negated propositions of non-classical logic, as for instance in the first period of the celebrated 1927 paper.

> We believe we understand the physical content of a theory when we can see its qualitative experimental consequences in all simple cases and when at the same time we have checked that the application of the theory never contains contradiction. (Heisenberg 1927, p. 62) (here and in the following emphasis is added in or-

> der to make apparent the two negative words of a doubly negated proposition) (Heisenberg 1927, p. 63).

In the first part of the same paper some more doubly negated statements are easily recognized; yet they do not play crucial roles in the development of the paper. The author reasons by means of non-classical logic – i.e. an *ad absurdum* argument – also when requiring the observability:

> … one must specify definite experiments with whose help one plans to measure the "position of the electron", <u>otherwise</u> this word has <u>no</u> meaning (Heisenberg 1927, p. 64).

More important is the fact that the entire development is aimed to obtain an *analogy* with the classical description:

> All concepts which can be used in classical theory for a description of a mechanical system can also be defined exactly for atomic processes in analogy to classical concepts (Heisenberg 1927, p. 68).

This kind of organization, a PO, was confirmed by subsequent Born and Jordan's papers.

In sum, both choices of Heisenberg formulation appear to be the alternative ones. Heisenberg was aware of the distance of his theoretical framework from the classical one, whose reference formulation was Newton's theory. In the introduction of his 1927 paper he underlines that in the change from the classical to the new framework some magnitudes undergo radical variations in meaning or even in the existence. Yet, he did not explicitly name Newton; worst, he founds QM on the magnitudes defining a state in the classical Newtonian framework, i.e. the position and then the velocity. Hence, his formulation is not consistently founded on the alternative couple of choices.

Rather, the subsequent Bohr-Jordan's paper made use of more adequate variables, i.e. the generalized p, q and the Hamiltonian function, which is essentially the energy of the system, i.e. the basic magnitude of the Carnotian theories. No surprise if a comparative analysis of the two papers gave very different basic assumptions. (Fedak, Prentis 2009, pp. 135-136).

However, Beller stressed that the original program remained incomplete: "Born and Heisenberg abandoned their original program" (Beller 1983, p. 475). Hence no definite formulation was achieved.

At present, after Born's introduction of the state function, the matrix mechanics is considered as a picture, precisely that picture in which the operators (observables and others) incorporate a dependence on time, but the state vectors are time independent, i.e. an arbitrary fixed basis rigidly underlying the theory. It represents an opposition of a complementary kind, to Schrödinger's picture, in which the operators are constant and the state evolves in time. All that corresponds to the difference between the active and the passive transformations. The Heisenberg picture is the formulation of matrix mechanics in an arbitrary basis, in which the Hamiltonian is not necessarily diagonal. But this correspondence with Schrödinger equation depends from the Stone-von Neumann theorem about the limiting space of functions; at glance, this limit appears a non constructive one. Hence this formulation appears to be unconcluded.

3. The re-formulation of matrix mechanics by T.F. Jordan

Thomas Jordan attempted to reformulate in a radically simpler way the previous formulation. His basic idea was to simplify the basic notion of this formulation, i.e. the matrix, which, by requiring cumbersome calculations, was antipathetic to the theoretical physicists. By exploiting the fact that at present the spin is a common notion and moreover its matrices (i.e. Pauli matrices) are usual, Jordan starts from these notions for illustrating the basic of his formulation, i.e. the notion of probability, the basic rules of the spin and magnetic moment and other quantities, the measurement process, the uncertainty relation. With this machinery, not so much difficult, he can present the applications of QM to a series of cases: quantized oscillator, Bohr's model of atom, hydrogen atom, small rotations and even changes in location, time and velocity. This list is overlapping with the list of successful cases of the original theory; see Beller (1983, p. 487).

The structure of the formulation, entirely devoted to some applications, is the alternative one, i.e. a PO. In fact, its basic problem is clearly how overcome the indeterminacy relation, to which the author gives emphasis both in its historical origin and in its theoretical role. Moreover, Jordan exploits the Hamiltonian, in which the commutation rules correspond to the commutators in the classical framework. In such a way, he can tackle by means of a simple theoretical procedure a series of important applications without calculus, so that this formulation can be presented to even high schools students. Hence, it is obvious that Jordan's mathematics is the constructive one (PI).

Yet, this formulation meets three basic criticisms. In the case the variables are continuous, the applications of his formalism are bounded to intervals of these variables. The finite operators cannot achieve, by a limit process, the continuum. This point was underlined by a reviewer (Jagannatan 1986). Second, since the theoretical apparatus is so centred on the simplification of the matrix formalism, the theoretical development, requiring of course more than the simpler matrices, is disregarded and a complete theoretical framework is missed. Third, he does not make use of doubly negated propositions.

4. Bub's informational quantum mechanics

This author claimed that his formulation is organized as a "principle theory" (in Einstein's sense). Beyond this analogy, one can show (Drago 2014) that its choice is the PO for three reasons. It is based on a problem, the quantum measurement. Its principles are methodological principles and are expressed as doubly negated propositions, i.e. they belong to the intuitionist logic. His principle (iii) is proved by an *ad absurdum* theorem. Moreover, it appears to be bounded to use the constructive mathematics (PI) for two reasons. Its mathematics, being related to information theory, is constructive. In addition, it is located in a C* algebra with bounded operators; this boundedness assures the constructivity (Pour-El & Richards 1989).

However, also this formulation meets a radical criticism. Being the information a non-physical magnitude, the entire formulation may be considered an artful.

5. Conclusion

The previous analysis of the three formulations of QM relying upon the alternative couple of basic choices shows that no one of them is consistently based on this couple (the case of matrix mechanics) or is completely developed (the case of Jordan formulation) or is free of a basic criticism to be an artful (the case of Bub formulation). Hence an alternative formulation of QM has to be still suggested. The basic choices suggest the foundations, but its theoretical development has to be still invented.

Appendix: The analogical reasoning and the correspondence principle as evidence for a reasoning in intuitionist logic

The analogy suggested by Heisenberg deserves attention. Heisenberg's 1927 paper at its beginnings underlines the radical variations in meaning of the basic notions, a variation so much radical to make insecure any traditional notion. Hence, there is no more sure propositions to be analytically investigated, and hence no more deductive path of arguing. Moreover, theoretical physicists had to tackle an at all unknown microscopic world, where the new phenomena and new concepts were so surprising that both the traditional intuition and the traditional conceptual framework could no more preserve as absurd the classical absurd facts. Hence, also a basic argument belonging to the context of the discovery was missed, the *ad absurdum* argument.

In the time of the beginnings of QM, the only way of reasoning in a PO theory remained to produce analogies, those more closely as possible to both known facts and classical physics. Notice that an analogy is a doubly negated proposition: "It is <u>not</u> true that it is <u>not</u> the case…". Hence, unwarily quantum theorists reasoned in intuitionist logic. Hence, theoretical physicists had to reason not only through new concepts, but, very much more importantly, *inside a truncated logic; which, even worse, was the unknown intuitionist logic*.

In his celebrated 1905 paper on quanta first Einstein put an analogy – between a particles gas and a quanta gas – as the aim of his theory.[1] Then was Bohr's analogy of

[1] The crucial roles played by the analogy and the correspondence principle in the beginning period of QM was underlined by a book (Darrigol 1992). He accurately studies three cases: i) Planck radiation theory; 2) Bohr atom and Heisenberg matrix mechanics; iii) Dirac quantum mechanics. He qualified Planck's case as "not a quantum discontinuity" and hence, in my opinion, it is improperly included in the list. Rather he dismissed Einstein's paper, which properly the first paper on quantum mechanics and was based on an analogy (Drago 2014).

the atom as a solar system, which played a decisive role, notwithstanding it is incorrect with respect to the indeterminacy principle. In 1924 De Broglie was addressed by an analogy to state that "material particles in motion behave as waves and wavelength associated with material particle is given by $\lambda = h/mv$". Matrix mechanics was constructed by an analogy that it had to be "as close to the mechanics of classical theories as could reasonably be hoped" (Heisenberg *et al.* 1926, p. 322), Even Schrödinger's equation was suggested by an analogy with the classical differential equation of the mathematical Physics; so much that for a long time he tried to characterize in material terms the current described by this equation.

The basic fact was illustrated by Jammer:

> Despite its high-sounding name and its successful solutions of numerous problems in atomic physics, quantum theory, and especially the quantum theory of polyelectronic systems, prior of 1925, was, from the methodological point of view, a lamentable hodgepodge of hypotheses, principles, theorems and computational recipes rather than a logical consistent theory. Every single quantum theoretic problem had to be solved first in terms of classical physics; its classical solution had then to pass through the mysterious sieve of the quantum conditions or, as it happened in the majority of cases, the classical solution had to be translated into the language of quanta in conformance with the correspondence principle. Usually, the process of find "the correct solution" was a matter of skilful guessing and intuition rather than of deductive and systematic reasoning. In fact quantum theory became the subject of a special craftsmanship or even artistic technique, which was cultivated at the highest possible degree of perfection in Göttingen and Copenhagen. In short, quantum theory still lacked two essential characteristics of a full-fledged scientific theory, conceptual autonomy and logical consistency…
>
> This discrepancy [between orbital frequencies of a atom and classical frequencies] was smoothed over by Bohr's heuristically invaluable principle of correspondence (Jammer 1966).

In 1913 this general practice was promoted by Bohr to a basic principle, i.e. the *correspondence principle*. A historian of physics summarizes what the important historian Max Jammer wrote about it (Bokulich 2010):[2]

> "[T]here was rarely in the history of physics a comprehensive theory which owed so much to one principle as quantum mechanics owed to Bohr's correspondence principle" (Jammer 1966, p. 118). The correspondence principle not only played a pivotal role in the discovery of quantum mechanics but was also the cornerstone of Bohr's philosophical interpretation of quantum mechanics, being closely tied to his better known thesis of complementarity and to the Copenhagen interpretation…

[2] In this paper the history, the historical interpretations and the present interpretations of the correspondence principle are listed and illustrated.

According to Jammer, the correspondence principle, interpreted as the frequency relation, applies by fiat to all quantum numbers and hence obtains the status of a "principle," even though it is an "approximate" relation that is only exact for large quantum numbers.

Jammer is rather dismissive of Bohr's claim that the correspondence principle should be thought of as a law of quantum theory. He writes,

For taking resort to classical physics in order to establish quantum-theoretic predictions, or in other words, constructing a theory whose corroboration depends on premises which conflict with the substance of the theory, is of course a serious inconsistency from the logical point of view. Being fully aware of this difficulty, Bohr attempted repeatedly to show that "the correspondence principle must be regarded purely as a law of the quantum theory". (Jammer 1966, p. 116)

On Jammer's view, Bohr's claim that the correspondence principle is a law is simply an attempt to cover up the inconsistent foundations of the old quantum theory. In opposition to Bohr's claim that quantum theory is a rational generalization of classical mechanics, Jammer interprets Bohr as viewing quantum and classical mechanics as irreconcilable, and hence interprets the correspondence principle as only a "formal analogy of heuristic value." At the end of his discussion of the correspondence principle, Jammer concludes "his [Bohr's] numerous and often somewhat conflicting statements, made from 1920 to 1961, on the essence of the correspondence principle make it difficult, if not impossible, to ascribe to Bohr a clear-cut unvarying conception of the principle" (Jammer 1966, p. 117).

I conclude that the analogy was the basic way of rationally arguing of quantum theoretical physicists before the achievement of the final formulation of QM. Surprisingly, never a similar collective theoretical behaviour occurred in the previous history of physics – not even in the special relativity –, and, even more surprisingly, its logical causes have been ignored, as well as its belonging to intuitionist logic.

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