On the concept of degenerate stars: the case of white dwarfs

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Abstract: In this work we briefly review the history of degenerate stars from the first introduction of Fermi-Dirac quantum statistics to the first unified approach of white dwarfs, based on the relativistic generalization of the Feynman-Metropolis-Teller of compressed atoms, which takes into account consistently the gravitational, the weak, the strong and the electromagnetic interactions.

Keywords: History of astrophysics, Compact stars, Degenerate Fermi gas

1. Prologue: Adams and Eddington

Exactly one century ago Walter S. Adams published a paper entitled "The Spectrum of the Companion of Sirius" in which he concludes that:

> … the companion of Sirius has a color index not appreciably different from that of the principal star (Adams 1915, p. 237).

This conclusion would lead to a dramatic consequence as summarized by Arthur S. Eddington:

> … a ton of [companion of Sirius] material would be a little nugget that you could put in a match-box. What reply can one make to such a message? The reply which most of us made in 1914 was - 'Shut up. Don't talk nonsense' (Eddington 1927, p. 50).

Nine years later, in 1924, Eddington published a paper entitled "On the Relation between the Masses and Luminosities of the Stars" in which he suggested a new test to confirm (or reject) the exceptionally high density of the companion of Sirius:

> … the question could probably be settled by measuring Einstein shift of the spectrum… (Eddington 1924, p. 322).

In 1925 Adams, following the suggestion of Eddington, published a paper entitled "The Relativity Displacement of The Spectral Lines in the Companion of Sirius" (Adams 1925) in which he

> … killed two birds with one stone. He has carried out a new test of Einstein's general theory of relativity, and he has shown that matter at least 2,000 times denser than platinum is not only possible but actually exists in the stellar universe (Eddington 1927, p. 52).

2. Fowler

After Adams results and Eddington theoretical considerations the existence of such compact stars, called white dwarfs by Willem J. Luyten (1922, p. 357), became one of the major puzzles of astrophysics. An important turning point came out on December of 1926 when Ralph H. Fowler published a paper entitled "On Dense Matter" in which the fundamental connection between the energy and the temperature in such high matter density systems is investigated (Fowler 1926). In particular Fowler wrote:

> … The excessive densities involved suggest that the most exact of statistical mechanics must be used to discuss the relationship between the energy, temperature and density of the material. This is a form suggested by the properties of atoms and the new quantum mechanics, which has been already applied to simple gases by Fermi and Dirac (Fowler 1926, p. 115).

Fowler explicitly refers to two papers published respectively on May of 1926 by Enrico Fermi (1926) and on August of 1926 by Paul A. M. Dirac (1926).

Fowler then shown that the equilibrium of very dense matter in white dwarf stars depends on the so-called *degeneracy* formula in which the electron pressure is related to the power 5/3 of the electron number density: the concept of degenerate star was born (Fowler 1926, p. 121).

3. Thomas and Fermi

Fowler introduced the Fermi-Dirac statistics to describe the interiors of dense stars pointing out the concept of degenerate star which

> … is strictly analogous to one gigantic molecule in its lowest quantum state. We may call the temperature than zero (Fowler 1926, p. 115).

Fowler however did not attempt to determine the equilibrium properties of such a star apparently because he was unaware that on November 1926 Llewellyn H. Thomas had sent a paper to the Royal Society in which he developed a method to solve this problem for a complex atom (Thomas 1927). Further on December 1927 at the "Accademia dei Lincei" in Rome was presented a note in which Fermi developed the same idea of Thomas (Fermi 1927). The method is a notable simple approximation of the Schrödinger theory of many electron atom (after termed as Thomas-Fermi model), which works particularly well in the case of compressed atoms i.e. degenerate matter at high densities.

4. Frenkel

The first application of the Thomas-Fermi model in order to describe degenerate matter at high densities was done by Y. I. Frenkel in a paper entitled “Anwendung der Pauli-Fermischen Elektronengastheorie auf das Problem der Kohäsionkräfte” (“Application of the Fermi-Pauli Theory of Electron Gas to the Problem of Binding Forces”) published in 1928 (Frenkel 1928) where he describes the dense matter in degenerate stars as arranged in a lattice where the elementary cell can be treated as a globally neutral Wigner-Seitz cell filled of degenerate electrons. Then developing a traditional Thomas-Fermi model for an atom Frenkel gives the Coulomb corrections with respect to the uniform electron distribution. He straightforward predicts two types of degenerate stars which consist respectively of the non-relativistic and ultra-relativistic degenerate electron gas and the stars of the second type should be massive, i.e. $M \gtrsim M_\odot$.

5. Stoner and Anderson

Following the work of Fowler (and ignoring the work of Frenkel) Edmund C. Stoner in a paper entitled “The Limiting Density in White Dwarfs” published in 1929 gives the equilibrium properties of white dwarfs (such as masses and radii) in the non relativistic limit for electrons, assuming a constant matter density approximation (Stoner 1929). Wilhelm Anderson in a paper published in 1929 entitled “Über die Grenzdichte der Materie und der Energie” (“On the Limiting Density of Matter and Energy”) noticed that in a white dwarf with a mass comparable to or higher than a solar mass electrons become relativistic, re-obtaining one of the conclusions of Frenkel (Anderson 1929). Stoner, following the *suggestion* of Anderson, in 1930 published a new paper entitled “The Equilibrium of White Dwarfs” in which, introducing the effect of special relativity, discovered the critical mass of white dwarfs (Stoner 1930).

6. Chandrasekhar and Landau

Following Stoner’s work, Subrahmanyan Chandrasekhar, in a paper published in 1931 and entitled “The Highly Collapsed Configurations of a Stellar Mass” (Chandrasekhar 1931), pointed out the relevance of describing white dwarfs by using an approach initiated by Edward A. Milne (Milne 1930) of using the mathematical method of the solutions of the Lane-Emden polytropic equations (Emden 1907). The same idea of using the Lane-Emden equations taking into account the special relativistic effects to the equilibrium of stellar matter for a degenerate system of fermions, came independently to Lev D. Landau (Landau 1932). Both the Chandrasekhar and Landau treatments were clear in pointing out the existence of the critical mass.

7. Kothari, Feynman-Metropolis-Teller, Wheeler-Harrison-Wakano

In 1938 Daulat S. Kothari published an article entitled “The Theory of Pressure-Ionization and its Applications” rediscovering the Frenkel approach (Kothari 1938). In 1949 Richard P. Feynman, Nicholas C. Metropolis and Edward Teller published an article, entitled “Equations of State of Elements Based on the Generalized Fermi-Thomas Theory”, where the correct application of the Thomas-Fermi model is considered in order to describe the equation of state of matter at high pressures and at various temperatures without and with the exchange terms (Feynman *et al.* 1949). In 1958 John A. Wheeler, Kent B. Harrison and Masami Wakano incorporated in the equation of state for degenerate matter at high density the inverse beta decay (Wheeler *et al.* 1958).

8. Salpeter and Hamada

Edwin E. Salpeter in 1961 published a paper entitled “Energy and Pressure of a Zero-Temperature Plasma” in which all of the elements introduced separately by Kothari in 1938, Feynman, Metropolis and Teller in 1949, Wheeler, Harrison and Wakano in 1958, are simultaneously present, in order to describe the equation of state of dense matter at zero temperature (Salpeter 1961). The article is followed by a second one (with Tetsuo Hamada) entitled “Models for Zero-Temperature Stars” where the equation of state derived in the preceding paper is applied to derive the equilibrium configurations of white dwarfs in the Newtonian limit (Hamada 1961). In particular here it is found that the critical mass of white dwarfs is a function of the chemical composition of stars and that this mass occurs, because of the inverse beta decays, at finite central density.

9. White dwarfs in general relativity

The authors mentioned in the above sections performed the analysis of the equilibrium of the internal structure of white dwarfs within the Newtonian theory of gravity. Nevertheless, the first treatment of white dwarfs in general relativity goes back to 1949 when Samuil A. Kaplan published an article, entitled “Sverkhplotnyè Zvezdy” (“Superdense Stars”), where it is shown that the general relativity induces a dynamical instability of a white dwarf when its radius become smaller than $1.1 \cdot 10^3$ km (Kaplan 1949). The general relativistic instability for white dwarfs was rediscovered by Chandrasekhar in 1964 (Chandrasekhar 1964). In 1968, using the criterion given by Chandrasekhar, Annie Baglin claims that:

> … the dynamical instability is reached very much before the instability due to the electron capture… (Baglin 1968, p.143).

In the same year Craig J. Wheeler, Carl J. Hansen and John P. Cox published a paper entitled “General Relativistic Instability in White Dwarfs” in which they conclude that:

> … the actual demise of the star will not be clearly be attributable to either electron capture or general relativistic instability but may be due to the nearly simultaneous onset of both effects (Wheeler *et al.* 1968, p. 255).

In that time the stability of white dwarfs near the Stoner-Chandrasekhar-Landau limit remains an open problem.

10. The necessity of a self-consistent approach

From 1968 onward several papers on the equation of state for degenerate matter in general relativity have been published.[1] However the various adopted equations of state present inconsistencies (Rotondo *et al.* 2011a). In order to avoid these inconsistencies a new approach based on the relativistic generalization of the Feynman, Metropolis, Teller equation of state has been developed (Rotondo *et al.* 2011b). From a theoretical physics point of view, this is the first unified approach of white dwarfs taking into account consistently the gravitational, the weak, the strong and the electromagnetic interactions and it answers open theoretical physics issues in this matter. No analytic formula for the critical mass of white dwarfs can be derived and, on the contrary, the critical mass can be obtained only through the numerical integration of the general relativistic equations of equilibrium together with the relativistic Feynman-Metropolis-Teller equation of state. New equilibrium configurations are obtained leading to the possibility of a direct confrontation with observations especially in the case of low mass white dwarfs. Further the theoretical base presented establishes also the correct framework for the formalism of the more general cases when uniformly rotations, magnetic fields and finite temperatures are present (Boshkayev *et al*. 2013a, Boshkayev *et al*. 2013b, de Carvahlo *et al*. 2014).

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[1] See, e.g.. (Rotondo *et al.* 2011a) and references therein.

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