Epistemology of harmonics

Danilo Capecchi - Sapienza Università di Roma - danilo.capecchi@uniroma1.it

Abstract: Ancient harmonics is the theory of organization of notes with respect to highness or lowness of their pitches. It was a mathematical science, whose deep knowledge was not necessary for a professional musician and thus was cultivated essentially by mathematicians and philosophers. They considered harmonics as a mixed mathematics, where the mixing was between experimental assertions derived from hearing and propositions derived from reason, that is mathematics. This classification was maintained through Classic and Hellenistic Greece, Middle Ages and Renaissance. Notwithstanding the important role played by harmonics in ancient science, historians have given little consideration to it. In this paper it is shown as harmonics merits a careful study and that its methods are the same of other sciences. Attention is devoted both to Ptolemy conceptions who used for harmonics the same approach employed in his astronomy, astrology, and geography and Renaissance harmonics that was a crucial role in the epistemology of the new science.

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1. The science of harmonics in ancient Greece

The theory of organization of notes with respect to the highness or lowness of their pitches was named harmonics by Greeks. Harmonics was a mathematical science, thorough knowledge of which was not necessary for a professional musician and was thus cultivated essentially by mathematicians and philosophers. There were two main traditions, usually referred to as Pythagorean and Aristoxenian. The former assumed that pitches of notes could be represented by integers and their relations by ratios (usually simple ratios; this was the dominant position of music theorists, namely Philolao, Archytas of Tarentum, Plato, Euclid, Theon, Nicomachus, Ptolemy). The latter denied the fundamentality of the mathematical representation of the pitches and developed an essentially empirical approach, albeit quantitative and for this reason still flanked by mathematics.

Greek mathematicians and philosophers considered harmonics as a mixed mathematics (Renaissance terminology), the mix being between experimental assertions, derived from hearing, and propositions, derived from reason, that is mathematics. This classification was maintained throughout the Middle Ages, where harmonics was part of the quadrivium (arithmetic, geometry, music and astronomy), and the Renaissance

(Palisca 1985). In any scientific discussion of the 17th century, harmonics was involved. Despite the important role of harmonics in ancient science, historians of science have given little consideration to it.

Greeks did not use the concept of an absolute value for the pitch of a note; they only could say that one pitch was higher than another (today we know that pitches are associated with the frequencies of vibration of air, a physical magnitude measured in Hz – cycles per second. A note corresponds to a given frequency; for instance we associate 440 Hz with the note La). In a Greek context a note alone had no sense, it was "irrational" (Ptolemy 2000, p.15). A note thus should be associated with other different notes. They are termed *concordant* if create a homogeneous impression on the hearing, *discordant* if do not (Ptolemy 2000, p. 17). Instead of the actual musical instruments, for their experimental analysis the scholars of harmonics made use of the monochord, which was simply a vibrating string whose length could be varied. The number expressing the length of the monochord was associated to the note it produced. The ratios of length defined the intervals among notes. Typical ratios of concordant notes were 2:1 (octave), 4:3 (fourth), 3:2 (fifth).

2. Ptolemy's *Harmonica*

Claudio Ptolemy (c.100 AD - c.175 AD) in his treatise *Harmonica* (Ptolemy 2000) resumed the positions of his predecessors. Though he could be classified as a Euclidean, he gave to experience a very high role.

Harmonica is composed of three books each divided into sixteen chapters. References concerning epistemology are scattered throughout the treatise. From the very first pages, it is quite clear that Ptolemy's efforts indicate that his epistemology is not only about music but about all of the science. However, in the *Harmonica* the epistemological positions – surprising for us today – are fully explicit, more than they were for example in the *Almagestum*, which could appear to be a much more scientific text. Ptolemy sought an agreement between a mathematical theory, made explicit by a number of assumptions on numerical relationships, and experience, which concerns the consent of the human ear to sounds, in the sense that it finds them somehow melodious. Ptolemy was convinced that this agreement should exist because nature itself has an intrinsic harmony. It is man's purpose to recognize this through the use of reason and senses.

The roles of reasons and hearing are distinct: «hearing is criterion for matter and condition, while reason is the criterion for form and causes» (Ptolemy 2000, p. 3) Ptolemy specified that the role of hearing is «discovering what is approximate and accepting what is exact», while the role of reason is «accepting what is approximate and discovering what is exact». That is, given two or more sounds, the role of hearing is to discover the approximate ratio of their pitches for which the sounds appear concordant. Then reason examines the ratios and corrects them if they appear irrational, that is contrasting with the hypothesis of harmonics. Subsequently these rationally modified ratios and mathematical inference from them are subject to hearing that could/should give its assent. Ptolemy specified that sense perception is characterized by incertitude. In any case there is a limit to human perception. But a repeated hearing can help perceive even very small differences.

2.1. Ptolemy's musical hypotheses

Ptolemy defined his musical hypotheses stating how they differed for those of the Pythagoreans and Aristoxenians. Those regarding Pythagoreans are of greater interest and only these will be remarked on.

Ptolemy said that their rational criteria were quite satisfactory, but they failed in some aspects; in particular they were not able to furnish rational criteria for all concords; for instance they could not justify the concord. Thus he suggested his own criterion, which actually was not very different from the Pytagoreans'. To the fully rational criteria Ptolemy added another of an empiricist nature.

Ptolemy was not troubled about mixing rational and empiricist principles; his epistemology permitted this, organized as it was in two distinct phases. The first inductive phase forwards the hypotheses. Here Ptolemy is completely free in his choice, which could even be made by chance – though this would be quite a modern approach. So any source could be assumed for the hypotheses made. In the second deductive phase theorems are derived from the hypotheses, assumed as principles of a *more geometric* argument. Theorems consist in propositions to be verified by hearing. If consent is gained the hypotheses are validated independently of how they were obtained.

3. Renaissance harmonics and acoustics

It is a matter of fact that in the 16th and 17th centuries there was a great involvement of mathematicians on problems of music or better harmonics: René Descartes, Marin Mersenne, Johannes Kepler, Christiaan Huygens, Athanasius Kirker, John Wallis. Galileo Galilei considered several fundamental musical questions in his scientific writings and was not only a lutenist himself, but a son, brother, and father of musicians, in short, a member of a musical dynasty (Palisca 1961, p. 92). All of them rejected the old numerological concepts of Pythagorean origin, and looked for new solutions to the age-old riddle of consonances. All of them adhered to the Copernican theory.

3.1. Physical approach

In the past – Ptolemy *docet* – harmonics was a mixed mathematics based on pure mathematical (both geometrical and arithmetical) basis, but neglecting many aspects that could be classified as physical, that is demanding causal explanations about generation and perception of sounds. While in the 16th century the main problem still concerned how to solve the problem of consonances, tuning and temperament, in the 17th

century, with the emergence of instrumental music, attention was focused on problems associated to new instruments, and studies on pipes also assumed relevance.

There were questions whose answers were not considered satisfactory by all. Why concordance shall be expressed as simple relations between numbers and why exactly by those numbers? How is man able to perceive a harmony in what might be only noise? To answer these questions, harmonics had to abandon the purely mathematical interpretation of phenomena and begin to explore the field of acoustics, a discipline that still belonged to the natural philosophy.

The study of acoustic leaded to a recognition of the laws for predicting pitches of vibrating strings, a quantitative theory of sound in air, a certain comprehension of the overtones and their superposition, the comprehension of the phenomenon of beats. Following the trend emerging in other mathematical-physical disciplines, acoustics was studied by mathematicians taking into account both causal explanations – characteristics of natural philosophy, particularly mechanistic – and experimentations. Experimentation, however, in many cases, did not assume for mathematicians the role that had in Ptolemy's harmonics – testing of a hypothesis –, but rather highlight new phenomena.

Vincenzo Galilei (c.1520-1591), Galileo's father, pursued a severe criticism to the Euclidean approach. He was a professional theoretical musician, whose fame rests on his pioneer working in the Fiorentine Camerata, a group of musicians who provided the beginning of Baroque music. In his *Dialogo della musica antica et moderna* (Galilei 1581) he launched a fierce attack to his master Zarlino. What is interesting from our point of view is that Galilei rejected the pure mathematical rationalist approach and asserted that there were no reasons to consider as natural those consonances that are in simple ratio. He was not convinced by an explanation based on numerology and that the series of simple numbers 1, 2, 3, ... it not the only one which could explain concordances. Everything depends on the quantities that are measured. In the case of the monochord the natural sequence 1, 2, 3, ... is the one associated to the lengths of the strings. But if one considers strings of the same length but subject to different force of tension, for example due to different weights, tone registers concordances for weights represented by the sequence 1^2, 2^2, 3^2, ...

3.1.1. Giovanni Battista Benedetti

Giovanni Battista Benedetti (1530-1590), in two letters written around 1563 to the musician Cipriano de Rore (c.1515-1565), afforded the problem of consonances on physical basis. The letters were published in the *Diversarum speculationum mathematicarum et physicarum liber* of 1585 (Benedetti 1585, pp. 277-283).

Benedetti drew his conclusions on the basis of various elements, meanwhile from empirical observation, then by his vocation toward mixed mathematical descriptions and then taking into account his knowledge of the natural philosophy of the time; in particular the conceptions of sound propagation. They were not very clear and probably not very original; essentially he believed that the vibration of a string transmitted their motion to the air, with a succession of phases of maximum movement and stillness, which were perceived by the ear. The different vibration frequency characterizes a note.

Benedetti's main contribution involved the formulation of the mathematical law according to which the frequency of vibration of a string is inversely proportional to its length. His argument was however not based on a true demonstration. Benedetti also succinctly wrote something that explains, or at least that can be interpreted as an explanation of, the phenomenon of consonance referring to frequency vibration. Addressing in particular to the eighth, he pointed out that in every two vibrations of the shorter string, the longer one is in concordance with it: «will concur or percuss.» The reasoning is repeated for the fifth; so the phenomenon of consonance is explained by the concurrence of maxima for the vibration of air of the two strings. The circumstance that the consonance is explained with the periodic correspondence of vibrations is known as the *correspondence law*.

3.1.2. Isaac Beeckman

There are doubts that Benedetti's writings arrived to the ears of Vincenzo Galilei, but they for sure came to those of Beeckman who read *Diversarum speculationum mathematicarum et physicarum liber* in 1633. He commented with appreciation on many sections, but Benedetti's two letters to de Rore were not among the passages quoted. He resumed the coincidence theory for the concordance, proving also what for Benedetti was a hypothesis, that is the inverse proportionality between length and frequency of vibration.

Beeckman's theory of sound is not simple also because it changed in time. Essentially Beeckman conceived sound as due to corpuscles (atoms?). Any vibrating source cuts the surrounding air into little (spherical) corpuscles that are sent away in any direction. When the corpuscles reach the sense of hearing they give raise to the heard sounds: «Sound [in the ear] is the way air was in the mouth of the speaker» (Beeckman 1939, vol. 1, p. 92).

In a first phase the different characteristics of notes, pitch, loudness, were explained by Beeckman respectively by the frequency of vibration – defined by the size of globes – and the quantity of air struck. Colour of tones was instead unexplained. Later on he explained pitches with speed and tone colour with the size of globes. In the first assumption the ratio of pitches corresponding to consonances is justified by the ratio of the size of globes. For instance the diameter of globes in an octave is 2:1. When assuming the second kind of explanation of the pitches, the consonance is associated to the contemporary presence in the vibrating air of an instant of rest, an explanation similar to that of Benedetti. For Beeckman the only pure consonance is the unison, all other consonances consist of a mixture of unison, when the speed of air is zero for both sounds, and dissonance, when the speed is zero in one case and maximum in the other.

3.2. Mersenne law for frequencies of vibrating strings

The quantitative dependence of the frequency of vibrating strings as function of the string characteristics, length, size, force of tension, is commonly named *Mersenne law*. Indeed although it was already known by others scholars, it was Mersenne that verified it through detailed experiments and diffused it. Mersenne referred to his founding in his studies of musical arguments that were collected mainly in two impressive texts, the *Harmonie universelle* of 1636-1637 with a total of about 1500 dense pages (Mersenne 1636), and the *Harmonicorum liber* of 1635-1636 of about 400 pages. By the end of the 16th century there was a great variety of musical instruments like the organ, the lute, the viola, the spinet, etc. which arose great interest in mathematicians and in Mersenne in particular.

Mersenne law was largely experimental; exception is for the dependence on the length that is 'proved' analytically. Mersenne resumed and perfected Beeckman's reasoning, without mentioning it explicitly. Unlike Galilei, Mersenne believed to be important the determination of the actual value of the frequency, in cycles per second, and carried out direct measurements of frequency – counting the oscillations – in the case of a very long rope.

3.3. Overtones and harmonics

Another interesting phenomenon known at Mersenne's time was the presence, empirically verified, of more notes in the sounds emitted by the various instruments including string instruments (presence of overtones) and the possibility of the presence of only notes higher than usual one (harmonic notes), which will be referred to as the fundamental note. Mersenne took care only of the first aspect that was well known since ancient times. The problem appeared difficult to explain because once the pitches were associated to the frequencies, the presence of multiple pitches simultaneously seemed to require that a body (a string) could vibrate simultaneously with multiple frequencies, and this was at least strange.

The possibility to hear only pitches higher than the fundamental ones is a much more complex phenomenon to explain. It was known in the late 17th century by some musicians and became the object of study by mathematicians. Experimental and theoretical analysis showed that the shape of a vibrating string could be in general much more complex than the 'parabolic' one commonly admitted. In particular there might be intermediate points of the string that are at rest. The possibility that there were intermediate fixed points, reported since the 18th century as *nodes*, was brought out first, experimentally, by John Wallis in 1677 (Truesdell 1960, pp. 118-120); but the author who first made a serious analysis of the harmonics was Joseph Sauveur (1653-1716) at the end of the 17th century. He introduced the terms *harmonic sounds* (or *harmonics*) among other things, for frequencies multiple of the fundamental one, and *ventres* (antinodes) and *nodes*, respectively for points of maximum and zero amplitude. He also contributed to spread the term *acoustics* to indicate the science of sounds in general (Sauveur 1701, p. 299).

4. Conclusions

Harmonics had been an important mixed mathematical science from Classical and Hellenistic Greece, throughout the Middle Ages and Renaissance. Despite this, however, it has been largely ignored by historians. This paper shows that harmonics merits a careful study and that its methods are the same as that of other sciences, such as astronomy or mechanics, to name but a few. Harmonics, perhaps, was the only scientific discipline where experience consisted of real experiments, very similar to those carried out in modern laboratories. Traditional harmonics provided an interesting rationalization key of musical compositions. But there were questions whose answers were not considered satisfactory by everyone. To answer these questions, harmonics had to abandon the purely mathematical interpretation of phenomena and begin to explore the field of acoustics, a discipline that until the 17th century belonged to the natural philosophy.

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