Archimedes between tradition and innovation

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Abstract: This paper contributes to the critical analysis of ancient scientific thought, specifically that of Archimedes. Continuing with the interpretative position adopted in previous work, we analyse excerpts from *De Planorum Aequilibriis* and *De Corporibus Fluitantibus*. The Archimedean concept of κέντρα των βαρέων is compared to the existing suspension point of a balance. The metaphorical nature of scientific terms used by Archimedes is confirmed in other excerpts and we outline how, from this perspective, Archimedes appears as a brilliant innovator working inside a pre-existing paradigm. A paradigm that in the following centuries would essentially be abandoned.

Keywords: History of science, Hellenistic science, Archimedes.

Introduction

Since the second half of the 19th century numerous critical analyses have contributed to a progressive reorientation in the interpretation of Hellenistic science and in particular regarding the figure of Archimedes.

The concept of *Hellenism* introduced by Johann Gustav Droysen (Droysen 1836) had already highlighted the peculiarity of Alexandrian culture with respect to Ancient Greece in general. This, however, was largely ignored in the history of science until well after the second half of the last century, and Hellenistic science continued to be read in function of the two great philosophical systems of the 4th century: Platonism and Aristotelianism.

Plato's influence on Greek mathematics was questioned by Otto Neugebauer (Neugebauer 1969) and, later on, by Knorr who, taking a much more radical position, interpreted the entire production of Greek scientific thought as the practice of problem solving (Knorr 1978). This, he claims, would have been carried out with little consideration of underlying philosophical issues. Sebetai Unguru (1975; 1979) criticizes the ahistorical conception of mathematics which sees mathematics as a given objective discipline and therefore independent of language, while arguing instead for the need to recuperate its historical collocation. He observed how ancient texts were presented using modern algebraic language under the presumption that the content would remain unaltered. To address this issue Unguru, followed by Raviel Netz (2004a) and others, suggested the literal translation of ancient texts. It should be noted, however, that a purely literal rendition of a text is not in itself enough to recover its historical collocation, nor does it address in any way the interpretative issues.

For example Frajese's translation is openly presented as a literal translation, however, leaving aside any eventual philological questions, in the accompanying notes many traditional prejudices are re-proposed. Among these, in particular, is Archimedes' presumed adhesion to Platonistic concepts.

Lucio Russo's (1997) reading, on the other hand, while radically breaking from tradition, perhaps oversimplifies the real historical process and offers a static and deterministic view of science which is still over attached to the positivist vision. Basing his analysis on a net demarcation of what he considers to be *science*, Russo concludes that it was the result of a rapid revolution that took place at the beginning of the 3rd Century BC and rapidly disappeared with the rise of Roman culture.

We have examined and tried to clarify other aspects of this question in previous papers (Migliorato, Gentile 2005; Gentile, Migliorato 2008; Migliorato 2005; Migliorato 2013a; Migliorato 2013b). Above all, however, it is about embracing the view of science as a process that develops over time, and as such, it is produced by collectively organized activities which interact within the whole socio-political and cultural context.

In other words, the definition of that which can legitimately be called scientific is not merely a question of demarcation, rather it is the search for a connecting thread winding through history, that allows us to collocate cultural phenomena, even those of different eras, into similar categories.

Archimedes, from this point of view, cannot have failed to understand that he himself was operating within a tradition, or scientific paradigm. In the 3rd century BC this appeared to be solid and deduced from the Euclidian texts, the only notable mathematical texts prior to Archimedes to have survived up to the present. This is why in our previous work the term *Euclidian paradigm* is used in this context.

It is significant that researchers in other fields have hypothesized a tradition of thought passing from Pythagoras to Democritus to Eudoxus up to Archimedes rather than to Plato and Aristotle.

However, we believe we can demonstrate with sufficient clarity how, starting in the 1st century BC that tradition, the previously termed scientific Euclidian paradigm, was subjected to a progressive obfuscation eventually becoming illegible while, at the same time, there was a corresponding increase in Aristotelian thought and the neo-Platonic school flourished.[1] This is the starting point of the interpretative tradition which did not cease even when the recovery and progressive evolution of scientific paradigms had started.

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[1] In Migliorato (2013a; 2013b) we show how the difficulty of reading a scientific text is determined by the same technical language in which the text itself is expressed. It is a language that is generally passed on directly from the master to the disciple. If this relationship is interrupted even for only a few generations it can quickly become incomprehensible or its meaning may alter. The dramatic events that besieged the Hellenist world, and not only in relation to Roman expansionism, could explain such a process.

2. Archimedes and his time

In the following pages we wish to examine more closely and contribute further analysis to previously mentioned research.[2] An essential analysis, we believe, given the decidedly divergent range of opinions that continues to exist.[3] On the one hand, supporters of the Knorr school consider the question to be out-dated, while others continue to see an inclination towards Platonism in Archimedes or at least they distance him radically from scholars of his time. On this last point Raviel Netz exemplifies:

> In most of Archimedes' letters there is a faint note of exasperation: there was no one to write to, no reader good enough. (There would be, in time: Archimedes would eventually be read by Omar Khayyam, Leonardo da Vinci, Galileo and Newton: these were Archimedes' real readers and the ones through whom he made his real impact. He must have known that he was writing for posterity) (Netz, Nöel 2007, p. 40).

The reasons leading Netz to this conclusion were discussed in a previous paper (Migliorato 2013b).[4] We will limit ourselves here to observing how Archimedes developed, in reality, all his work inside a technical-conceptual apparatus that was largely pre-existing and shared even when new and more advanced prospects were on the horizon. His careful attention to sharing, as we will see, is constant and there is no hint of the "mystic visionary". This aspect would be more evident had his pronouncements been made outside the shared language. The reflections of the neo-Platonic Proclus are of a different nature as highlighted in the following brief example:

> It is evident to everyone that the equilateral is the most beautiful of triangles and most akin to the circle, which has all its lines from the centre equal and a simple single line bounding it from without. And the enclosing of the triangle by the two circles, by each of them indeed only in part – for it is inscribed in the whole of neither circle but only in the area consisting of segments of both – seems to indicate in a likeness how the things that proceed from first principles receive perfection […]. And if, furthermore, every soul proceeds from Nous and returns to Nous and participates in Nous in a twofold fashion, for this reason also it would be proper that the triangle, which is a symbol of the three natures in the constitution of the soul, should take its origin from being comprehended by two circles.[5]

Is it chance, or as we believe, is there an intrinsic incapability in terms of geometric results, if Proclus' contribution amounts to zero whereas in our daily experience we observe the new results that even the most modest researcher obtains when operating within the paradigm in which he or she was trained?

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[2] It should be noted that Migliorato (2013b) cited here was unfinished at the time of presentation of this paper to the XXXIII SISFA Congress.

[3] In this respect the observation of an anonymous referee from a prestigious international journal joins the choir of voices that claims no serious scholar today would sustain the existence of Platonic or Aristotelian influences on Archimedes. The opposite view was offered, however, by another anonymous referee of the same journal, who considered the revisionist hypothesis to be too bold.

[4] See note 2.

[5] Proclus, *Comm. Elementi*, 215. Trasl. by Glenn R. Morrow (Proclus, Morrow 1970, p. 167).

The greatness of Archimedes should be seen not only in his extraordinary ability to obtain results but also in his re-interpretation of the existing paradigm, which he enriched with new potential and meaning.

The fundamental paradigm, pre-existing, shared and based on the work of Euclid has certain characteristics which for reasons of space we will summarise[6] as follows: *use of undefined technical terms denoting abstract entities; absence of pronouncements of a metaphysical and ontological nature regarding the truth of hypotheses or the existence of objects designated by technical terms; a posteriori criteria of validity for the assumption of hypotheses and postulates.*

The last prerequisite shows that Archimedes' reasoning in the acceptance of an assumption never refers to a more or less evident *a priori* truth, but rather is derived from an acceptance of the results obtained, from the ability to supply solutions to otherwise unsolved problems, or to problems which had previously been resolved in an unsatisfactory or approximate way, and, in the final analysis, by general consensus which also included the experts.

We will return to this question (*a posteriori* validation) later locating further instances to add to those highlighted elsewhere. For reasons of space here is a very brief description.

3. Conjectures and refutations

We will start with a passage from *De Quadratura Parabola* in which Archimedes, remembering previous attempts by geometers to find the area of a circle and other curved figures, says:

> Some of those who worked earlier in geometry attempted to prove (draw) how it would be possible to find a rectilinear area equal to a circle that's given or a segment of a circle that's given, and afterwards they attempted to square the area enclosed by the section of the cone as a whole by assuming some lemmas that were not readily admissible (ούκ εύπαραχώρητα λήμματα). For this reason, these were condemned by most people as not being discovered by them.[7]

It is not difficult to see here Archimedes' reference to the criticisms raised against Antiphon and Bryson regarding their attempts to find the area of a circle.[8] John Philoponus[9] for example offers two interpretations of Bryson's attempts, one from Alexander of Aphrodisias and the other from Proclus. In the former the area can be found using the following scheme: given a circle, it is possible to inscribe a polygon and circumscribe another circle; furthermore, it is possible to construct another polygon

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[6] See our previously cited work for more details.
[7] Archimedes, *Quadr. Parabola* (Archimedes, Mendell 2013).
[8] Aristotle, *An. Post.* 75b37-76a3; *Ph.* 185a14-17; *Re. Sof.* 171b12-22, 171b34-172a13. We do not know if Aristotle was the only one or was, at least, the first to raise such criticisms. It is certain, however, that the lemma proposed by Bryson as the basis of his proof had been challenged and was therefore considered.
[9] Philoponus, *Commentary on Aristotle's Prior Analytics* 75b37.

between the two previous ones and this polygon and the circle are in the middle of every polygon inscribed and every polygon circumscribed. At this point Bryson assumes that “things which are bigger and smaller of the same thing are equal to each other” and so can conclude that the polygon and the circle are equal. Proclus on the other hand reconstructs the area of a circle as follows: the circle is bigger than every inscribed polygon and smaller than every circumscribed polygon; Bryson assumes that “where there is a smaller and a bigger, there is also an equal” and, since there is both a smaller and a bigger polygonal figure than the circle, he can conclude that there is a polygonal figure equal to the circle.[10]

We will not examine the well-known terms of Aristotle’s[11] criticisms but will follow Philoponus’ interpretation, which, as well as offering a counter-example,[12] permits us to identify more clearly the demonstrative scheme under accusation. Philoponus observes how Bryson does not “construct” a polygon that contains all the polygons inscribed and is contained in all the polygons that are circumscribed, nor does he offer a method to effect such a construct. In essence you must presume the existence of such a polygon to be able to prove its equality with the circle. We can also say that since for every pair of inscribed-circumscribed polygons there exists another polygon in the middle of the two, we cannot conclude that a polygon exists between any pair of inscribed-circumscribed polygons. In modern terms this is the same as saying that from the proposition: $\forall x, \exists y: (P(x) \Rightarrow Q(x,y))$ it doesn’t necessarily follow that $\exists y: \forall x\ (P(x) \Rightarrow Q(x,y))$. This is just one of the errors in the history of mathematics and remnants can be found even in Cauchy, for example, in the proposition which affirms that the limit of a converging series of continuous functions is itself continuous. As Lakatos has shown (1976), it was Cauchy who gave the first demonstration of this conjecture and throughout the 18th century its truth was so entrenched and widely accepted that it seemed to have no need of proof.

The discovery of counter-examples led to a careful critique of the demonstrations offered by Cauchy finding the solution by assuming a property that is not implicit in the simple convergence and is subsequently called *uniform convergence*. Lakatos considers the creation of uniform convergence as an incumbent correction to the conjecture.

We wanted to recall this modern case because we feel that Lakatos’ analysis offers insights that are useful to our argument. Lakatos calls definitions *generated-from-demonstration* those, as in this case, that are created to save enunciations that would otherwise prove to be erroneous. Lakatos favours this explanation of the growth of mathematical knowledge calling it the *heuristic approach*, as opposed to the strictly *deductionist approach*. In other words, according to Lakatos, the growth of mathematical

[10] Since every polygon can be squared, the area of a circle is obtained in both cases.
[11] Aristotle, *An. Post.*, A, 9, 75 b 37-76 a 3.
[12] Philoponus also reports what today would be called a counter-example when he mentions that the angle of contact (the angle between the arc of the circle and the tangential) is smaller than every rectilinear angle and the angle of the semi-circle (the angle complementary to the angle of contact) is superior to every rectilinear angle; since therefore a rectilinear angle is greater than every angle of contact and smaller than every angle of the semi-circle it can be concluded that there is no rectilinear angle equal to a contact angle (or to an angle of the semi-circle). Philoponus explains this depends on the fact that segment and arc are two different things and thus he finds an instance when Bryson’s assumption does not hold.

knowledge, would not (or would not always) occur by following a strictly deductionist approach as in starting with *a priori* premises as would be expected, rather it follows a much more complex route. Beginning with conjectures and intuitions, scholars then try to provide proof, they are often confronted with counter-examples which are gradually eliminated by adding conditions to the original enunciation, conditions that validate the conjecture or in, any case, resolve the question.

Naturally the deductive structure remains the basis of validity for all the solutions found and it is into this structure that they must be placed to receive their definitive form. We could say that the real problem on which the mathematicians often find themselves confronting, even if it appear in a different form, seems to be, according to Lakatos' interpretation, that of incorporating conjectured solutions through the formulation of *lemmata* and additional hypotheses working within the deductive structure.

4. *Mechanical methods*. Towards a new paradigm?

Without ascribing an absolute value or giving a thorough evaluation of Lakatos' interpretation, we believe, however, it allows us to clarify at least one aspect of the process of the growth of science, not only in the modern age but also, as far as possible, in the time of Archimedes.

In the previous work cited above we examine other instances which support this view and have been further examined (Migliorato 2013b). Among these the concept of *barycentre* is of particular interest. Leaving aside the question of the "lost book",[13] we highlight how this concept is not reasonably definable in realistic terms while, at the same time, its accompanying postulates are sufficient for its characterization within the scientific context. If, however, we examine its genesis, we can see that the concept is presented as a metaphorical and abstract translation or, rather, as a shift in meaning of the pre-existing concept *centre of suspension*. Its eventual assumption as a technical term and its characterization through postulates within the scientific context is justified therefore on the strength of the results obtained by using it and the problems it has managed to resolve.

The eminently geometric characterization of the concept of barycentre in *On the Equilibrium of Planes* is outlined in Migliorato (2013b). This characteristic is clearly established in postulate 4 where the expression "centre of weight" appears for the first time. It affirms that "if two figures are exactly superimposed their respective barycentres coincide". It is evident that here, given the geometric configuration, no other extra-geometric fact can modify the barycentre. Later on the concept itself is extended to the more general context of "size" without further specification thus permitting its use in mechanics (*On Floating Bodies*). A series of steps during which shifts in meaning occur within a fixed technical apparatus of assumptions and postulates.

[13] This refers to the hypothesis of a lost work of Archimedes which contained an explicit definition of barycentre (literally in Archimedes κέντρον τού βάρεως or *center of the weights*). See for example (Sato 1981). A reasoned refusal of this hypothesis is contained in Gentile, Migliorato (2008) and Migliorato (2013b).

In itself it is not surprising therefore that Archimedes uses it in a purely geometric environment, when this can be done without problems. We find it again in *On the Quadrature of the Parabola* but its use becomes problematic when consideration of an infinite totality is implied. The method of exhaustion, in this respect, appears to be suitable here where it is used to circumvent the problem of infinity by bringing the solution back to a finitistic process. However, its fruitful use is not automatic or simple and this must have induced Archimedes to further contemplation and research. This is enough perhaps to explain the scientist's need to raise objections and challenge his contemporaries on a series of problems that he himself encountered.

The *mechanical* procedures based on the concepts of *suspension* (virtual) and *barycentre* that are outlined in *The Method* appear to be more suitable and offer a more direct and general course leading to results. However, in doing this, Archimedes claims, the results obtained still cannot be proven. An affirmation that has always been interpreted as a refusal to adopt into geometry concepts that were borrowed from mechanics. This conclusion, however, is not completely convincing and, on closer inspection and on the basis of accepted postulates, nothing exists in those concepts that is not geometrically determined. Even the term *equilibrium* itself is susceptible to the loss of its original *mechanical* connotations when it is seen as a precise and specific state of geometric configuration in reference to a determined point. The question of infinity, we believe, is much more meaningful.[14]

As Fabio Acerbi has pointed out in his introduction to *The Method*, nearly all the assumptions made by Archimedes in this work were based on theorems which had already been demonstrated, with the exception of two. These two introduce something new and challenging:

> Lemma 3. If the centres of gravity of any number of magnitudes whatever be on the same straight line, the centre of gravity of the magnitude made up of all of them will be on the same straight line.
> Lemma 4**.** The centre of gravity of any straight line is the point of bisection of the straight line (Archimedes, Heath 1912, p. 14).

Taken individually, they may seem innocent enough but the effect of combining them is that: *if in a plane figure we consider all the straight lines that are parallel to a given straight line, then Lemma 3 can be applied to it*, thus becoming, for the first time in Hellenistic geometry, an enunciation about an infinite totality.

We will try, at this point, to draw some conclusions, all be they provisory and in no way conclusive. The true personal beliefs of the Syracuse scientist can be argued however one views the more or less legendary figure presented by tradition. Was Archimedes a Platonist? What ideas did he had about the world, about its effective reality, about its knowableness? We don't believe this query can ever be answered

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[14] The question of Archimedes and infinity has been dealt with in other papers (Migliorato 2013b). The search to extract as much as possible from the indeterminacy of infinity in all its possible determinations and not just infinity per se, can be found in several parts of the Archimedean opera. This was in fact a pre-existing tendency in Hellenistic mathematics and we can find it already in Euclid, whose enunciation of the 5th postulate constitutes, in this sense, an absolute masterpiece (see Migliorato, Gentile 2005; Migliorato 2013a).

satisfactorily and, while it is licit to discuss and hypothesize upon it, we are interested in another aspect. We are concerned with analysing Archimedes in relation to his "public" work as a scientist. His role, in other words, in the collective process that, as mentioned in the introduction, is viewed as an organism, emerging from the complexity of history. We have analysed his work from this position and our final considerations are so related.

In the introductory letters to his works Archimedes does not at any point allude to principles of absolute "truth" or mention proof. He tends to validate his own assumptions by relying upon the trust already awarded them by predecessors and by their extensive use in problem solving. It is his hope that theories based on these assumptions will be likewise accepted in the future. These are all elements of a pre-existing shared scientific paradigm.

Archimedes, however, goes on to develop further this paradigm, he operates within it to construct new conceptual entities and widen its apparatus. He does it creatively, producing shifts in meaning and making new assumptions and throughout it is accompanied by a need to justify every procedure, every assumption so that it can be followed, and made use of by others.

The Method is no exception and Archimedes expresses his hopes that others will put its contents to good use. However, *The Method* contains something more: the conceptual leap contained in Lemmas 3 and 4 taken together that presupposes the existence of an infinite totality. A conceptual leap which, up until then, had been steadily avoided and could not therefore belong to, nor could it automatically be included in, a shared paradigm.

This does not imply that such a paradigm could not be assumed in the future, nor do we know if Archimedes was aware of this or not. He does not say, as neither does Gauss who avoided publication of his conclusions on the 5th postulate and on the possibility of a non-Euclidian geometry for fear of the "clamour of the Boeotians" (Reid 1970, p. 59).

In fact the new paradigm did not emerge immediately. Every hypothesis regarding what could have happened, if the flourishing spring of Hellenistic science had continued, belongs to the realm of conjecture and, as such, it cannot be regarded as history.

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