The explicit use of symmetry as a principle. Study of the symmetry notion as a metalanguage term in relativistic physics

Ruth Castillo - Universidad Central de Venezuela - chebichev@gmail.com, ruthcastillo@usb.ve

Abstract: Symmetry is for physics what is understood as conservation laws. It is natural for physicists today to derive laws of nature and prove their validity by means of laws of invariance or conservation instead of deriving these laws from those we believe are the laws of nature. This turn represents the first turning point in the application and use of the notion of symmetry in twentieth-century physics as a metalinguistic term. Thus, the magnitudes are automorphisms, ensuring the invariance or conservation of laws in any reference system, showing symmetry as a metalinguistic term. In this article, we postulate the explicit use of symmetry as a principle, studying this notion as a metalanguage term in relativistic physics, assuming that, under certain transformations, the aspects that characterize phenomena, systems or laws are unchangeable, thus being independent from any particular observation (principles of symmetry).

Keywords: Symmetry, Principle, Metalanguage, Invariance, Physics.

1. Introduction

Symmetry is a fundamental notion in physics, showing different meanings: a) Heuristically, it models the search for satisfactory solutions to different problems under a series of statements (for example, the qualitative descriptions of the ancient world warranted equilibrium and harmony observed in the world) and b) Methodologically at present, the theories are studied as structures. The reasons are: 1) The evidence provided by the history of science, 2) The terms acquire their meaning from the theory, and 3) The progress of the theories is more efficient if it contains within them prescriptions on what to do for the advance. From the methodological meaning, in the seminal paper (Mainzer 1990) the notion of *symmetry* presents an important ontological and epistemological question: the problem of whether symmetrical structures are human inventions or a structure of real principles that organizes and determines the world (some believe in an ontological reality of symmetrical structures independent of human models and ideas). From a methodological point of view, currently the ontological question of symmetry is discussed. However, despite the dilemma, we show that a description of nature in terms of symmetric structures and symmetry breaks seems to be the appropriate way to describe the diversity and complexity of reality.

Under this perspective, for Krauss (2007, p. 188) *symmetry* is a notion in current physics that embraces conservation laws. It is natural for physicists to derive laws of nature and prove their validity by means of laws of invariance or conservation, instead of deriving these laws from what we believe are the laws of nature. In this sense Kastrup (1987), argued that Einstein pointed out this inverted approach when he postulated the universality of the space-time continuous as a consequence of Noether's theorems, which represented the first turning point in the explicit use of the notion of symmetry to twentieth-century physicists. Under the previous perspective, through the algebraic language used in physical theories, we postulate *symmetry* as a metalanguage term revealing the explicit use of the notion as a *principle* in relativistic physics. Weyl (1958) argued that, the magnitudes are *automorphisms*, which ensure the invariance or conservation of laws in any reference system, which show *symmetry* as a metalinguistic term. In this research, we explore the explicit use of symmetry as a *principle*, in terms of Roche (1987), under the study of the notion as a metalanguage term in relativistic physics, assuming that under certain transformations the aspects that characterize phenomena, systems or laws are unchangeable, thus being independent from any particular observation (*principle of symmetry*).

Krauss (2007) postulates the use of *symmetry* as a *principle* in contemporary physics that refers to conservation laws to validate the physical laws. Under this consideration, and taking into account, the use of the notion of *symmetry* as a *principle* in Noether's theorems: each symmetry within a physical system implies the conservation of some physical properties of the system at the same time that each conserved quantity corresponds to symmetry. In other words, the isometry of space accounts the linear conservation of momentum while the isometry of time accounts the conservation of energy. In the book (*La noción de Simetría en Física. Una reconstrucción*, 2008, pp. 107-108), we postulate that, the use of *symmetry* as a *principle* is the principle of relativity ensuring the invariance or conservation of physical laws at group of Lorentz and Poincaré transformations, while in general relativity the use of *symmetry* as a *principle* through the principle of general covariance. The validation of physical laws through the principle of invariance or conservation makes it possible to identify the explicit use of the notion of *symmetry* as a *principle* in relativistic physics. Now, physical theories include algebraic language. In algebraic terms, Weyl (1958) argued that spatio-temporal symmetry refers to aspects of space-time that exhibit a form of symmetry that complies with the properties of time and spatial translation, spatial rotation, Poincaré transformations and inversion transformations. Finally, we argue that symmetry connects empirical reality and mathematical structure through the language. The portentous mathematical apparatus requires understanding *symmetry* as a numerical function supporting the description of the world through the invariance of conservation laws under empirical transformations that are described in mathematical terms.

2. Invariance and Simultaneity

Kant (1928, p. 206) argues that things are simultaneous when the perception of subject *A* can follow the perception of subject *B* and vice versa. We can perceive first the moon and then the earth, or inversely, first the earth and then the moon. These successive perceptions are possible because the earth and the moon exist simultaneously. This reversibility in the order of perceptions is not possible in those successive phenomena. Thus, reversibility in the order of perceptions constitutes a subjective criterion, but this does not mean that simultaneity is derived from succession, since it is the temporal relationship (reversible or not) that we assign *a priori* what determines the subjective criterion of this temporal relationship. Simultaneity is defined by Kant (1928, p. 208) as "the existence of the multiple in the same time". This means that simultaneity does not have the same conception of time for Kant, in other words, it is not a pure intuition. Weyl (1958, p. 131) argued:

> Again, has the statement that two events occur at the same time (but in different places, here and in Syria for example) objective meaning? Until Einstein people said yes. The basis of this conviction is obviously people's habit of considering an event as happening at the moment when they observe it. But the foundation of this belief was long ago shattered by Olaf Roemer's discovery that light propagates not instantaneously but with finite velocity. Thus, one came to realize that in the four-dimensional continuum of space-time only the coincidence of two world points, "here-now", or their immediate vicinity has a directly verifiable meaning. But whether a stratification of this four-dimensional continuum in three-dimensional layers of simultaneity and a cross-fibration of one-dimensional fibers, the world-lines of points resting in space, describe objective features of the world's structure became doubtful.

From Weyl's statements, we can understand that simultaneity takes place when two events occur at the same point or very closely and at the same time. In other words, the time interval between two events or the distance between two points must be relative to the observer. Of course, the ideas of Weyl do not go in the same direction of the Kantian arguments. In terms of Weyl's relational character of simultaneity, two points very close to each other within a referential system ("here-now") evokes the space-time continuum, since spatial positions do not suffice for simultaneity: temporal simultaneity is required also. Two events will be simultaneous if they are relationally in the same (or very close) spatial and temporal point. Now, when are they not simultaneous? If we consider two different events in different positions, even if they are close to each other, one observer in each place, both within the same space-time, they are not necessarily witnessing the same event at the same time. So, how can we decide which description corresponds to reality? The relational perspective of Kant affirms that not all the observers will measure the same interval of time between two events or the same length for the same object, if they are not under an adequate relationship, in terms of Weyl in that "here-now". What happens with our theories? Should they be adapted to each particular reference system?

This relative character of simultaneity, observed by Kant and clarified in Weyl's ideas, demands, in physical terms, the descriptions made by different observers leave the laws of physics invariant. Thus, though two events are not simultaneous, or our observers do not perceive them at the same moment, their descriptions do not affect the validity or solidity of our theories or physical laws. In Physics is required that the measurements by the observers leave the theories or physical laws invariant in a determined group of transformations. In this way Castillo (2018, p. 73), independently that the events are simultaneous or not, the invariance is a fundamental property of the mathematical structures supporting the solidity of our physical laws. I emphasize, although each observer is in a different reference system and each one argues different descriptions or measurements, the physical laws or our theories remain invariant or unaltered. This requirement of invariance of physical laws in different systems of reference, argues that those descriptions, offered by two observers located at different points of space, should remain unchanged if they are invariant in terms of mathematical structures. Maintaining the structure of science requires laws or descriptions of the world to remain invariant and independent of the observer or reference system. Under these perspectives, Mainzer (1990, p. 319) argued that the notion of *symmetry* (understood in terms of invariance, conservation, equilibrium) evolves within current physics through mathematical language as *automorphisms*. The invariance of the laws of physics, in any inertial system, shows symmetry. Likewise, the invariance is a fundamental property in mathematical structures. The physical laws have been described through the mathematical structures, and the invariance of these structures refers to the permanence of the laws in each inertial system in a group of transformations.

3. The language of symmetry

Mainzer (1990, p. 319) argued that currently in physics, symmetry is understood explicitly as an automorphisms so, a transformation that preserves the structure of space. The automorphisms transform one figure into another by making them in Leibniz's terms, indiscernible, if considered separately. The term *automorphisms* is due to Leibniz, he argued that relations within space represent different transformations that leave the structure invariant (automorphisms). Weyl (1958, p. 46) argued "(1) Every figure is similar to itself; (2) if a figure F' is similar to F, then F is similar to F', and (3) if F is similar to F' and F' to F'' then F is similar to F". Mathematicians have adopted the word group to describe this situation, and so they say that automorphisms form groups".

Under these ideas, an automorphism or auto-mapping of figures leaves the structure invariant; likewise, automorphisms that fulfill these three conditions comprise in turn a group of transformations, which is plausible since they are a particular case of transformations. Thus, the reflection on a plane will be a transformation associated with bilateral symmetry. For example, in a balance in equilibrium, the exchange between any of its parts does not make the transformation distinguishable; this indistinction is known as reflection. Moreover, this transformation is an automorphism. In this way, iterating the identity (I) we will have an automorphism, since the identity (I) is an automorphism that applies each point p on itself. Let us expand a little this idea of identity as automorphisms. Weyl (1958, p. 45) exposes:

> Two applications S and T can be made one after the other: if S applies the dot p at p', and T the p' at p'' then the resulting application that we will call ST applies p at p''. An application S can have an inverse S' such that SS' = I and in turn S'S = I; in other words if S transports the arbitrary point p on p' then S' applies p' on p and the same condition must be satisfied if first S' and then S is carried out.

From the above, identity I is a transformation and will contain its own inverse. Unlike identity, two applications any ST does not have to be equal to TS, i.e. it does not have to be commutative. Therefore, automorphisms are particular transformations, although every automorphism is a transformation, not every transformation is an automorphism. In this sense, automorphism is a transformation that preserves the structure of space, so reflection on one plane is a basic operation of bilateral symmetry: from its iteration SS´ results identity (I), in other words it is its own inverse. Then this transformation (reflection) shows bilateral symmetry. Then, it´s possible to affirm that the group of transformations have automorphisms as subgroup, and these will contain a subgroup, the group of the congruences. The congruences can be understood as automorphisms that do not modify the dimensions of a body. The congruences in plane are the reflections and translations, referring to the bilateral symmetry, while the congruences in space as rotations, will respond to the spherical symmetry. Weyl (1958, p. 47) argued that the application of symmetry groups has been widely accepted in science, as it provides a language that explicitly describes symmetry in physical theories by showing the invariance of laws. Thus, in a reference system not only the points in space are represented numerically, also the physical magnitudes. Thus, the transformations between admissible reference systems leave the physical laws invariant, forming the group of physical automorphisms. We can understand physical automorphisms as congruent applications that, unlike geometric automorphisms, consider physical events in space and time.

Weyl (1958, p. 110) exposes: "(…) the world extends not as a three-dimensional continuum but as a four-dimensional continuum. The first to correctly describe the symmetry, relativity or homogeneity of this tetradimensional medium was Einstein". From the above, the structure of the physical world is revealed in general laws of nature; these laws are formulated through the magnitudes that, being functions of space and time, leave these laws invariant. These space-time functions are the physical automorphisms. In the case of simultaneity, the descriptions of both observers are within inertial systems, these descriptions or laws remain invariant because the systems are subjected to translations that exhibit symmetrical properties. The physical space-time (magnitude) or automorphism functions maintain the invariance in the physical laws.

4. Symmetry and its relation to conservation laws: Lorentz transformations and Noether's theorems

Honh and Goldstein (2008, p. 233) argued that A. Legendre established the term bilateral symmetry through geometric considerations rotating a figure many times until it is left in the same position, so it is not possible to know if the figure has been rotated or not. Similarly, it happens with reflections, dilations and translation movements. Such operations do not allow us to distinguish if such a figure was transformed. Therefore, this operations or transformations make indistinguishable the figure, not being able to assure that something has been applied to the figure. In the algebraic considerations, we can apply the same transformations to equations. Equations, have symmetrical properties (reflection, translation, dilation, etc.). The equations with these symmetric properties belong to the Galileo Group. The Galileo transformations (dilations, rotations and translations) are also contemplated by Newton. Hence, Newton's transformations are subordinated to those of Galileo.

The difference between the group of Lorentz transformations and the group of Galileo transformations lies in the fact that the former offers a transformation equation for time (taking into account the relative character of simultaneity) and consider the constancy of the speed of light. In this way, the Lorentz group leaves the spatio-temporal functions invariant, giving an account of the invariance of laws. Under this perspective, we can affirm that time is an automorphism because it is indistinguishable or indiscernible in a transformation. Time does not distinguish between past, present or future. For the magnitudes, it is indistinct left or right, up or down, today or tomorrow. Krauss (2007, p. 190) argued "a physical magnitude is a physical automorphism". In addition, the group of automorphisms contains as a subgroup the set of congruences, so physical automorphisms are defined as congruences, as we have said, the invariance of the laws against a transformation, in this case the Lorentz transformations. Then the physical automorphisms go on to account for the invariance in terms of the *equilibrium* (*equilibrium* in Greek sense, or *qualitative*).

Krauss (2007, p. 187) argued: "the symmetries of nature are responsible for guiding physicists in two important aspects: they limit the flow of possibilities and determine the appropriate way to describe the remaining ones". In the search for the description of the world, in science prevails the simplicity, preservation of equilibrium and invariance of any change (physical automorphism), in other words, symmetry. Any descriptive possibilities, which do not respond to these aspects, are reject in science. Then, through the invariance of physical laws with the consideration of physical automorphisms, the symmetry in nature is shown. In 1933, Emmy Noether analyzed this through her theorems. Krauss (2007, p. 190) affirms, that the physical automorphisms are expressed in laws and the laws contemplate equations that govern the behavior of a given system. A physical quantity that remains conserved or indiscernible – without preference to any spatial or temporal direction – and at the same time invariant in a transformation – physical automorphism – is expressed in physics as a conservation law. From this point of view, in physics when we talk about symmetry, we mean the conservation laws. In this way, a physical quantity indiscernible in terms of past and future, without preference in temporal directions and invariant in a transformation, is a physical magnitude conserved in time. In other words, the symmetrical properties of a physical system are intimately related to the conservation laws that characterize the system.

Krauss (2007, p. 192) affirms that Noether's theorem states that each symmetry of a physical system implies some physical property of the system is conserved. Each conserved quantity has a corresponding symmetry. Noether states this physical quantity is the energy independent of temporal directions. In this sense, Noether relates the physical magnitude of energy, through a conservation law, in other words: the isometry of time corresponds energy conservation. From these ideas of Krauss (2007, p.192), the energy is a conserved quantity as a consequence of symmetry in time – as Noether's theorem affirms – and in an analogous way the quantity conserved as a consequence of the symmetry of space will be momentum or inertia, that is: the isometry of space corresponds conservation of momentum. The conservation of momentum is the principle behind Newton's observation the objects will continue to move uniformly and those at rest remain in that state, unless an external force acts on them.

In the book *La noción de Simetría en Física. Una reconstrucción* (2008), we postulate: if two states are equivalent, the symmetry is shown through an invariant quantity (magnitude). For example, in the principle of inertia, the velocity (magnitude) is constant for rest and a uniform rectilinear motion. The indistinction between rest and uniform rectilinear motion for a local observer (review the classic Newton-Leibniz controversy across Samuel Clarke) show the symmetry of space. In other words, the speed as space-time magnitude is constant (physical automorphism) in a system at rest or in a uniform rectilinear motion (Galileo transformations) showing the symmetry of space.

5. Considerations

Finally, any *symmetry* in a physical system has its corresponding conservation law (and vice versa), constituting in this way an explanation of why there are laws of conservation and physical magnitudes (*physical automorphisms*) that do not change throughout the temporal evolution of a physical system. This argument is based in two basic ideas: (1) The invariance of the physical law with respect to any (generalized) transformation preserves the coordinate system (spatial and temporal aspects), (2) The conservation of a physical magnitude. In this way, the formal statement of Noether's theorems derives an expression for physical quantities conservation. Thus, (1) The invariance of physical systems with respect to translation movement is related with the law of conservation of momentum and (2) Invariance with respect to time is related with the law of conservation of energy. The result of Noether's work is far-reaching in any physical theory. It reduces everything to analyzing the various transformations that would make the form of the laws involved invariant. This important deduction, a consequence of the relativistic theory of Einstein, constitutes the turn in contemporary physics in relation to the consideration of the notion of symmetry as a *principle*.

References

Branding K., Castellani E. (2003). *Symmetries in Physics. Philosophical Reflections.* Cambridge: Cambridge University Press.

Casini P. (1971). *El universo máquina.* Barcelona: Martínez- Roca.

Castillo R. (2018). *La noción de Simetría en Física. Una reconstrucción.* Mauritius: Editorial Académica Española.

Frank A., Wolf K.B. (1992). *Symmetries in Physics,* Ciudad de Mexico: Springer-Verlag.

Geymonat L. (1970). *Filosofía y filosofía de la ciencia*. Barcelona: Labor.

Haywood S. (2011). *Symmetries and Conservation Laws in Particle Physics. An Introduction to Group Theory for Particle Physicist*. London: Imperial College Press.

Hon G., Goldstein B.R. (2008). *From Summetria to Symmetry. The Making of a Revolutionary Scientific Concept*. Cambridge: Springer Science.

Jammer M. (1970). *Conceptos de espacio.* Ciudad de Mexico: Grijalbo.

Kant I. (1928). *Crítica de la razón pura.* Translated by Morente M. Madrid: Librería General.

Kastrup H.A. (1987). *The contributions of Emmy Noether, Felix Klein and Sophus Lie to the Modern Concepts of Symmetries in Physical Systems*, in Doncel M. *et al* (eds.), *Symmetries in physics, 1600-1980. Proceedings of the 1st international meeting on the history of scientific ideas held at Sant Feliu de Guixols, Catalonia, Spain, September 20-26, 1983*. Bellaterra: Universidad Autònoma de Barcelona.

Krauss L. (2007). *Fear of Physics. A guide for the perplexed*. New York: Basic Books.

Roche J.J. (1987). *A critical study of symmetry in physics from Galileo to Newton*, in Doncel M. *et al* (eds.), *Symmetries in physics, 1600-1980. Proceedings of the 1st international meeting on the history of scientific ideas held at Sant Feliu de Guixols, Catalonia, Spain, September 20-26, 1983*. Bellaterra: Universidad Autònoma de Barcelona.

Mainzer K. (1990). "Symmetry in Philosophy and History of Science the Quarterly of the International Society for the Interdisciplinary Study of Symmetry". *Symmetry: Culture and Science*, 1 (3), pp. 319-328.

Weyl H. (1958). *La simetría*. Buenos Aires: Nueva Visión.