Symmetry or differential equations: adding a case study on conformal field theory

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Abstract: Symmetry and dynamics have been depicted as dichotomous methodologies in the work of physicists of finding theories to describe Nature. Many case studies, from Kepler laws to super-symmetry, have been analysed by Barut to characterize the two possibilities. A detailed nomenclature is proposed for the two methodologies by Drago, namely OP/IP and OA/IA. OP/IP stays for problematic organization and potential infinity, and refers to those theories based on symmetry principles. OA/IA stays for apodictic organization and actual infinity, and refers to theories based on differential equations, even if Drago's classification encompasses two dichotomies and not just one. Exercising the interplay among these foundational techniques is then thought as the methodological task of the theoretical physicist. To previous case studies those in conformal field theory (CFT) and related issues are added. Special attention is posed on the concept of "non-dominant thinking".

Keywords: symmetry, dynamics, dichotomies, conformal field theory, nonviolence.

1. Introduction

The main experiences this article is rooted in are the author's thesis with professor Ravanini (Amoruso 2016) on conformal field theory (CFT), professor Drago elaborations on fundamental readings about nonviolence (Drago 1995), and many of the talks of this SISFA conference.

In particular, I would like to draw attention on Rossi's talk *Death and resurrection of Field Theory: 1960-1975*. The basic element that makes this talk so interesting for the purpose of the present article is the individuation of a fundamental conflict between the field theoretical approach and the bootstrap approach in describing the same physics, namely particle physics. Each of these formulation of QFT dominated in different historical contexts, possibly repressing greater scientific cooperation. From this episode a moral could be drawn for physicists: exercise a non-dominant thinking, do not focus on too narrow formal assumptions.

A second talk, for some aspect similar to the first one, is Cerreta's *Mach principles of dynamics and Newton's bucket*, where again a conflict is present among Mach's formulation and Newton's formulation of mechanics, which, in this case, has led to the ideation of innovative didactical tools by Cerreta's team.

Thirdly, Favale, through his *The couple of Nasir al-Din al-Tusi*, turned on the interest of the audience on the al-Tusi engine, as conflicting with conventional engines. Here the conclusive open question was: why the al-Tusi invention is not used, despite its high efficiency?

What links these different contributions is the very presence of a conflict, or dichotomy, inside a given topics; this observation is in fact very general and in this context it concerns what could be called the cultural **nonviolence** of a scientific discourse. Nonviolence is a fundamental discovery for human spirit, both very ancient and very new; it asks to be manifested through every exterior action but roots deep in the very process of thinking. Given its interior basement it is not surprising that even scientific discourse can encase violence. This can take place at different levels, and all of them would require attention, though the focus of this topic is the level of "non-dominance" in thought. The works by Drago have the great merit to focus on the very mechanism of dominance in theoretical physics, from an historical perspective. It is on this point of view that we would like to pose our caring.

CFT is a highly peculiar research field where different mathematical techniques form a very rich and unusual theoretical *corpus*. This sector of physics lends itself to the mentioned kind of analysis that I would like to call "the Drago perspective", which consists in a critical scan of a given theory according to a newly proposed conceptual grid, as it will be shown in the following.

2. Two distinct methodologies in physics

2.1. The Barut perspective

The proper starting point of this paper is what could be called "the Barut perspective". It consists in the idea that two distinct methodologies exist in theoretical physics, namely symmetry and dynamics. Barut wrote:

> Symmetry and dynamics are different ways of formulating the laws of physics, not necessarily one derivable from the other; sometimes conflicting, sometimes complementary to each other, often answering to different types of questions, together necessary for a more complete understanding of nature (Barut 1986).

He also affirms that these methodologies are ubiquitous in the history of physics, though used with various levels of awareness. They have different characteristics indeed; first of all, they pose different questions. Symmetry asks "how is the world?", while dynamics "how does the world become?". Moreover they focus on different aspects of phenomena: symmetry grabs "global descriptions", while dynamics "local" ones. Also, they give different fruits – timeless "ratios laws" and "evolution laws" – through different mathematical tools: "algebra" and "analysis".

To exemplify, let us look at Barut's example on Kepler's laws. The dynamics-based approach would depart from the differential equation of motion, it would require a lengthy and skilful integration to finally obtain the orbit equation, which is Kepler's first law. From a symmetry-based approach it is instead possible to rapidly obtain Kepler's third law. From empirical observations of planet's speed on the *n*-th orbit,

$$v_n \simeq v_0 e^{\lambda n},$$

one could reconstruct the imaginary scale transformation among orbits in the space-time grid

$$x_n = e^{2\lambda n} x_0 \; t_n = e^{3\lambda n} t_0,$$

which maintains invariant the quantity

$$t^2/x^3.$$

This invariance coincides with Kepler's third law.

By the way, the idea of finding eternal ratios in the motion of planets was precisely the method of Kepler himself, even if he was looking for sacred platonic ratios, different from the ones he actually discovered and that we learn. Even if his method proved to be almost prophetic, dominant thoughts are always ready to discredit, inhabiting scientists of all times. In fact, Kepler's search has been defined as:

> "an idea so crazy by modern standards that it does not even make sense" (P.T. Mathews talking about Kepler search for sacred ratios in the motion of the planets, Inaugural Lecture, Imperial College 1962) (Barut 1986).

This quote represents an example of a dominant way of thinking.

2.2. The Drago perspective

Barut perspective finds an interesting extension and completion in what I would love to call the Drago perspective. It consists in a conceptual classification of the methodologies of theoretical physics, as can be studied in his works. This perspective individuates two fundamental dichotomies:

- a dichotomy on the infinite: IA-IP;
- a dichotomy on the organization of the theory: AO-PO.

Let us explain, recalling their meaning. IA-IP stays for "actual infinity"-"potential infinity", the continuous and the discrete; AO-PO stays for "apodictic organization"-"problematic organization", naming those theoretical structures grounded, respectively, in axioms or in some given problem. According to Drago perspective, this classification offers a new way to consider physical theories and the whole history of physics. In this way, particular emphasis is put on dominant thinking and alternative discourses in physics. The author is convinced that this original conceptual grid is actually fulfilling its purpose.

The Drago perspective is adopted for the first time regarding conformal field theory, in the following chapter.

3. Case study on CFT

Conformal Field Theory is essentially shaped by the Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n},$$

which determines the Hamiltonian, its eigenstates and the whole Hilbert space. Moreover, to each eigenstate is associated a field Φ, and the set of these fields respects the "Operator Product Expansion":

$$\Phi_i(z)\Phi_j(w) = \sum \frac{c_{ij}^k}{(z-w)^{h_i+h_j-h_k}} \Phi_k(w).$$

This serves to show how important algebra is in this field theory; according to Drago perspective one could then say that IP is dominant here. This is certainly true, but it does not suffice to describe the peculiarity of the internal structure of CFT. Indeed, according to Ravanini, CFT hosts exactly the meeting of algebra and analysis, of symmetry and dynamics. To show this compresence we report a formula to calculate commutators of operators, through (circular) integrals of complex analysis:

$$[A, B] = \int dw \int dz\, a(z)b(w).$$

Moreover, from the so-called fusion algebra, equations of motion and their Lagrangian can be derived:

$$\Phi_i \times \Phi_j = \sum N_{ij}^k \Phi_k \rightarrow \partial\bar{\partial}\phi = u[\phi],$$

showing the strong relationship among symmetry and dynamics.

For what concerns the organization of the theory, it can be said that the physical problem it deals with is the understanding and description of systems at criticality, which involves continuous phase transition in the context of statistical mechanics.

Single CFTs, though (i.e. CFTs where the value of the Virasoro charge *c* has been fixed) can be studied both through the bootstrap approach (that historically links to AO) and the field theoretic approach (PO). The famous (non-unitary) Lee-Yang model, for example, is known both via its S-matrix and its Hamiltonian:

$$S_{LY} = \frac{tanh\frac{1}{2}\left(\theta + i\frac{2\pi}{3}\right)}{tanh\frac{1}{2}\left(\theta - i\frac{2\pi}{3}\right)}, \qquad H_{LY} = \int dx \left[\frac{1}{2}(\partial\phi)^2 + i(h - h_c)\phi + ig\phi^3\right].$$

From this point of view, it can be said that, though strongly rooted in IP and OP, CFT contains a promiscuous body. In its internal structure different methods and terminologies have met, none of them dominating the others.

4. Conclusions

In this paper we have analysed CFT through the Drago perspective, showing elements of its foundational structure. It results in a very open and rich theoretical body, which makes contact with many different subjects in theoretical physics and in mathematics, and it has maybe been insofar studied less than it would deserve. The course taught by Ravanini at Bologna University is actually the first graduate course on CFT in Italy. Foundational dichotomies of the Drago perspective are assumed as useful categorisation in philosophy of physics and history of physics, in didactics and communication of physics, as well as in non-dominant thinking education; they are also considered possible sources for tales, reflection and research lines.

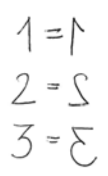

Fig. 1. Symmetric equalities

I would love to conclude this article with a personal story about non-dominant thinking, regarding one of my nephews. One day, while I was writing my thesis, he came close to me and asked what I was up to. I then tried to tell him about equations, basic operations and that sort of things, trying to express myself in a way appropriated to a six years old curious child. He listened to me, silently, and then hid away for a while. Then he returned with two little pieces of paper. On one of them there were some equalities, written in the strange way of Figure 1. He asked: “Is it right, uncle?”. He not only had understood equalities, but spontaneously rendered them symmetric, in his own way. Then he gave me the other one, saying: “Uncle, you said there is the minus, the plus, the equal… Is there this one as well? And this other one…?” (see Figure 2). He had also understood basic operations, but he was trying to generalize them. Then I happily renounced to any kind of prejudice about his thought, and I realized that I was having fun and learning something new. He was a very good teacher indeed.

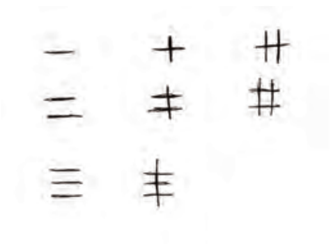

Fig. 2. Basic operators generalized

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