# Kant's philosophy of knowledge and the four models of a scientific theory

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*Abstract*: Along two centuries and half Kant's philosophy of knowledge received radical criticisms. Yet, to many scholars it appears as the less inadequate one to the study of the foundations of present science. These foundations have been recently represented by two dichotomies; according to which Kant's antinomies result a mistake of non-classical logic. The outcomes of the two dichotomies give four models of scientific theory (MSTs). They are compared with Kant's judgements. His philosophy of knowledge results to be i) essentially relying on the Newtonian MST; ii) a very far approximation of the four MSTs; iii) an echo of the novelties in his contemporary theoretical physics (Lagrangian mechanics). The last point is in agreement with Kitcher's recent analysis and gives reason of the persistence of a so controversial philosophy of science.

*Keywords*: Kant's philosophy of knowledge, Four models of scientific theory, Two basic dichotomies, Antinomies, Judgments.

## 1. Kant and the present philosophy of knowledge

It is custom to put at the top of the historical development of the philosophy of science Kant's system, because it is credited to have

a. conciliated the two main philosophical currents of the Western philosophy of knowledge – empirism and rationalism;
b. recognised the antinomies as a constraint for the reason looking for the "noumenos";
c. stressed the formal conditions under which a systematic thinking is possible;
d. suggested an interpretative scheme which conclusively interpreted the human knowledge.

However, few decades after its presentation, Kant's philosophy of knowledge (for short, Kpk) was disconfirmed by several new scientific theories; the birth of non-Euclidean geometries denied the *a priori* nature of Kant's transcendental category of space; the birth of thermodynamics denied two more categories, continuous time and causality; at last, the theoretical relevance of both thermodynamics and chemistry denied Kant's appraisal that the (more advanced) scientific theories have to make use of the (most advanced kinds of) mathematics (infinitesimal analysis). A century after him, a re-birth of logic as mathematical logic disconfirmed Kant's appraisal on Aristotelian logic as the most advanced logic as possible.

Therefore this philosophy received several criticisms stressing that he suggested as basic aspects of human knowledge some inaccurate and misleading notions. As a result, at the present time the various appraisals on Kpk range from the most optimistic ones to the destructive ones. Here is an example of the former ones:

> As far as the recent epistemology is concerned, the critical philosophy [by Kant] constitutes a theory of science that agrees with currents trends; for science must have a base that is empirical though also real. On the other hand the transcendental or a priori is implicated […] (de Vleeschauwer 1993, p. 475).

Some others say "Good-bye Kant".[1] At last: "there is no such thing as the standard interpretation of Kant's transcendental idealism" (Rohlf 2010).

On the other hand, Leibniz's philosophy of knowledge was criticised in a radical way as an essentially metaphysical one and eventually it was dismissed; moreover, Kant's philosophy of knowledge resulted to be more profound than the philosophies of science of the subsequent scholars. Neither a better interpretative scheme of science has been suggested by the recent historians of science, who interpreted modern science through naïve ideas; either a single notion (empirism, induction, causality, determinism, etc.), or two notions (Mach: the economy of thinking; Koyré: geometrization of space), or even six Kuhn's notions (which however are of narrative nature; moreover their accurateness was easily contested) (Mastermann 1970).

This point of advantage justifies the persistence of Kpk notwithstanding it accumulated so much denials by the subsequent development of science together with negative appraisals. After two centuries and a half Kant's basic ideas may stand as the most appropriate ones only *ad excludendum*.

> Despite a fundamental lack of clarity about the epistemic status of the 'critical philosophy' itself and penetrating absolute idealistic, realist and naturalist attacks on it, Kantianism influences almost all contemporary non-positivist philosophy of science as well as formalist and intuitionist philosophies of mathematics (Bhaskar 1981, p. 223).

These facts suggest that it is urgent to put an end to a so long period of uncertain and divergent appraisals on what is currently considered the top of the Western philosophy of knowledge. All in the above shows that a more radical or even drastic viewpoint is needed. By taking advantage from the much more long experience of theoretical science than Kant's one, I obtained a new viewpoint on the foundations of science

[1] This is the title of Ferraris (2013). For a review of past German interpretations of Kpk see Malter (1981). For negative appraisals see (Couturat 1905, pp. 235-238; Quine 1951; Beth 1966, pp. 39-47; Kitcher 1983). Beth (1959, p. ix) declared that his study (for his Ph.D.) of Kant's philosophy of knowledge deceived him. He rather referred the study of the foundations of modern science to Aristotle's model of deductive science. However he considered also it as overcome by the more recent scientific achievements, so that he advocated a radical change in the present viewpoint; but without suggesting a new proposal.

which is in agreement with some basic tenets of Leibniz's system. In the following I will compare it with two crucial issues of Kant's thinking, the antinomies and the judgements. Since it leaves aside much of his ideas the following is more than an interpretation of Kpk, a quick appraisal of some basic Kant's ideas.

## 2. The substantial inadequacy of Kant's philosophy of science with respect to his contemporary science

Recently, a scholar wrote "Kant's detailed theory of knowledge depends upon a now antiquated commitment to Euclidean Geometry, Newtonian mechanics and Aristotelian logic" (Bhaskar 1981, p. 223). Indeed, Kant pondered upon three scientific theories only: Aristotle's classical logic (by Kant conceived as a completed theory so that in all futures times it will not be changed), the Euclidean geometry (conceived by him as the only possible geometry) and the Newtonian mechanics-astronomy (conceived by him as the unique theoretical physics).[2]

Moreover, a century after the celebrated Leibniz's, Newton's and Euler's contributions to the birth of the infinitesimal analysis, Kant almost ignored its great influence on theoretical science. All modern scholars know that instead this mathematical invention originated in science an ontological metaphysics (in particular Euler's one (plus, in the subsequent centuries, the metaphysical commitment of both Rational Mechanics and Mathematical Physics). Owing to this ignorance, Kant may be considered as the last Greek of modern philosophy of science.

Moreover, Kant ignored all alternatives formulations of mechanics to Newton's.[3] As first, the attempts by Huygens and Leibniz to build a different mechanics; which was then accomplished by L. Carnot during Kant's life (Carnot 1783; Carnot 1803).[4] In addition, he almost ignored the theoretical relevance of the two celebrated foundations by respectively Maupertuis and D'Alembert. He also ignored that in his time the entire mechanics-astronomy was founded anew by Lagrange on the basis of the principle of virtual velocities; this theoretical improvement gave great scientific advancements (for instance, about the interpretation of the solar system). These facts lead to evaluate Kpk as essentially inadequate for a comprehensive interpretation of the varieties of his contemporary physical theories.

More in general, let us recall that in his time Paris collected half of all great scientists, that in a great number assumed high political responsibilities during the revolution; moreover, the French science (chemistry as first) played a revolutionary role in the society. Yet, Kant ignored the social role of science (Ben-David 1971; Gillispie 1959). He appears as a conservative aristocrat whose thinking relied on the Newtonian paradigm so that he rejected his contemporary French science.

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[2] Recall that at that time Newton's mechanics had included geometrical optics by conceiving a light beam as the trajectory of a massive particle.

[3] It is striking the great resemblance of Kant's definition of his absolute space with Newton's one. See the quotation in (Brunschvicg 1929, pp. 263-265).

[4] For a short presentation, see Drago (2004).

### 3. Kant's fallacy about the four antinomies

Kant suggested that in the human reason there exist some dichotomic oppositions which can be conciliated by neither the empiricists nor the rationalists; whatever intellectual path aiming to know the reality the human reason is pursuing, these dichotomic oppositions impede to achieve the "noumenos"; they are therefore declared antinomies.[5]

Kant's limitations to human reason played a very important role in stopping the pretension of some scientists, e.g. Euler,[6] who, by having built a marvellous scientific theory – as the new theoretical physics appeared at that time –, claimed that it was necessary to substitute for the old metaphysics a mechanics metaphysics.

To my knowledge, neither Kant nor anyone after him recalled that these dichotomic oppositions reiterate the two labyrinths that Leibniz saw in the human reason.[7] Truly, Kant criticised (Wolff's version of) Leibniz's philosophy of knowledge; but he declared that in 1768 the reading of Leibniz's Essays greatly impressed him (Kant 1798). It is likely that this reading originated "the great light" he received in 1769. Leibniz's two labyrinths of the human reason may have suggest to him similar basic categories.

In fact, Kant's four "antinomies" result from an application of the labyrinths. The two alternatives of each labyrinth are applied to specific notions of the metaphysical debate (space, time, infinity, subdivision) and presented as the thesis and the antithesis of a same subject. For both he suggested *ad absurdum* proofs. By comparing the two conclusions he stressed that one is the negation of the other one; hence, he declared them in contradiction.

Yet, the conclusion of each *ad absurdum* proof is no more than a doubly negated proposition. Only classical logic allows to derive from it the corresponding affirmative proposition. For instance the temporal antinomy gives the two conclusions:

1. It is *impossible* for an infinite world-series to have *passed away*;
2. *No* coming to be of a thing is possible in an *empty* time.

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[5] (Kant 1781). This claim was not a novelty; promptly Schopenhauer stressed that the inadequacy of the reason to the real world had already been suggested by Maupertuis and others (Cassirer 1906-1907, vol. II, book VII, chapter IV). In particular, Cusanus (Cassirer 1983, chapter 1) stressed that whereas the finite realm appears more easy to be grasped – owing to the several ways of (perceiving and) measuring it –, the infinite realm appears difficult to be understood, since our method to apperceive it is uncertain and disputable; but also all measurements on the finite realm are no more than approximate evaluations of reality; even less they can lead to a complete, objective knowledge of it. Hence, provided that a new method for merely approaching the infinity is established, no qualitative difference exists between the approximate knowledge of the finite realm and that of the infinite beings.

[6] See for instance, Euler's quotation in Cassirer (Cassirer 1906-1907, vol. II, book VII, chapter IV) and also the remarks by Brunschvicg (1929, pp. 275-276).

[7] He wrote: "two famous labyrinths where our reason very often goes astray." (Leibniz 1875-1890, vol. VI, p. 29). Certainly, Leibniz's vision of science was very profound: he was the last philosopher to be fully aware of the whole science of his time, since he gave a great contribution to all "hard" sciences; he founded (the beginnings of) the mathematical logic, the infinitesimal analysis, a new geometry (*analysis situs*) and (the beginnings of) a formulation of mechanics which is alternative to Newton's (Drago 2003).

In fact, Kant ignored any alternative to classical logic.[8] Being the operation of negation of the non-classical logic weaker than the classical negation, the conclusive predicates of the two proofs do not constitute a contradiction.[9] As a consequence, no antinomies result.

## 4. Interpretation of the origin of the two basic Kant's distinctions

Notwithstanding his "great destruction" operated by means of the antinomies, Kant had to suggest an interpretation of the current human capability to produce knowledge through e.g. science.

However, he "was enormously impressed by the discovery of these contradictions" (Kant 1798).[10] What did see Kant in these antinomies?

In fact, Kant's suggestions can be traced back to these fourfold structure of his antinomies and at last to the structure of the two dichotomic labyrinths, each allowing two outcomes. In fact he suggested several fourfold categories for interpreting various situations; e.g. the four categories in logic, the four categories of the human thinking, the four principles of phoronomy, dynamics, mechanics and phenomenology, etc.

Two centuries and half after Kant, a study of past scientific theories suggested a scheme of four models of scientific theory (MSTs) fashioned by the choices on two fundamental dichotomies, i.e. either the actual infinity (AI) or the potential infinity (PI), and either the Aristotelian deductive organization (AO), or the problem-based organization (PO. They substantially agree with Leibniz's two labyrinths.[11]

Did Kant anticipate by means of his own categories a similar structure? Unfortunately, all his fourfold scheme are almost disconnected;[12] moreover, no one of them perfectly agrees with the four MSTs, those for physical science too.

It is rather noteworthy that he interpreted the judgements given by the human reason by means of two independent distinctions, i.e. synthetic/analytic and *a priori*/empiric (or *a posteriori*). Are these distinctions similar to the two dichotomies originating the four MSTs?

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[8] Actually, he knew badly also the latter one (Kneale, Kneale 1962, V, 4). A drastic appraisal ("terrifying narrow-minded and mathematically trivial") is in (Hanna 2014, sect. 2. 1**).** Although no one empiricist remarked this point, in empiricism the (inductive) non-classical logic holds true; indeed, it is easy to recognise that it is possible to re-construct the main representative book of the empiricism (Hume 1748) according to the characteristic way of non-classical arguing of a scientific theory. This point is illustrated by (Drago 2012).

[9] This is the same conclusion obtained by Wundt (quoted by Couturat (1905, p. 301); and maintained by e.g. Enriques (1983, p. 77) from a different criticism; the three notions of finite, potential infinite, actual infinity have been at some extent interchanged by Kant.

[10] See also (Brunschvicg 1929, p. 274).

[11] About my conception of the foundations of science and its relation with Leibniz's system see for instance (Drago 1990, Drago 2013).

[12] A detailed criticism to Kant's classification of the four judgements is in (Kneeled, Kneeled 1962, V, 4).

According to Kant the word "analytic" means (see the following table) a purely tautological proposition; whereas Kant's definition for "synthetic" clearly refers to a proposition not obtained from a deductive theory.

However, commentators agree that these distinctions have been not clearly defined by Kant. First of all, he presupposed that every proposition is composed by a subject and a predicate. Instead, this supposition fails, for instance, when the subject is absurd or not existent; as a consequence, in theoretical physics the highly relevant principle of the impossibility of a perpetual motion is not a Kant's judgement. Also the mutual independence of Kant's definitions of the two distinctions is questioned, since in some sense they mutually overlap.[13] Hence, many kinds of correspondences between them and the two dichotomies are possible. Let us consider the correspondence that I consider as the most likelihood.

By taking seriously the two above Kant's distinctions, one has to expect that *four* cases will result. Instead, Kant considered *three* kinds of judgement only. He attributed the "synthetic a posteriori" judgement to the empiricism, considered by him as capable to produce knowledge at no more level than that of an art (as he stated chemistry's is).[14] The "analytic a priori" judgement was attributed by him to the metaphysicians. He then tried to conciliate these two philosophical currents by inventing a new judgement, i.e. the *synthetic a priori*. The fourth case, the *analytic a posteriori*, was discarded.

## 5. Interpreting Kant's judgements by means of the two basic dichotomies

Let us compare Kant's kinds of judgements with the four MSTs. Apparently his three-fold scheme cannot agree with the four MSTs. Instead it is possible to give reason for this difference.

Let us relate the distinction "analytic/synthetic" to the characteristic features of the propositions one meets in the two different kind of the organization of a theory, respectively AO and PO. Although the former distinction was mainly philosophical in nature, whereas the latter dichotomy is structural in nature, their main features are *grosso modo* in mutual agreement.

Moreover, the *a priori* may be put in correspondence with the great influence exerted on the entire science of his times by the infinitesimal analysis, since its basic notion, the infinitesimal, by representing directly AI, did not have a clear explanation and hence it has to be assumed as an at all a priori notion. In fact, till up to the end of the 19th Century several scientists intended this kind of mathematics as representing the best *a priori* instrument. On the other hand, they intended the words *a posteriori* [proved] as representing the opposite meaning, i.e. the operative way of making use of tools bounded to PI only; e.g. in Euclidean geometry the ruler and compass. I conclude

[13] See for instance, (Couturat 1905, p. 266, fn. 1; Quine 1950; Trudeau 1987, pp. 108-109; Kneale, Kneale 1962, V, sect.4; Walsh 1967, p. 309).

[14] For a long time a similar appraisal – to be a "phenomenological" science, or, as Kuhn stressed, a "Baconian" science, i.e. as essentially impeded to achieve a theoretical system – charged some theories, for ex. thermodynamics.

that Kant's distinction corresponds to the dichotomy on the kind of mathematics which was usual among his contemporary scientists, i.e. either the AI mathematics or the PI mathematics.

In the following table both Kant's four fundamental elements of the two distinctions and my interpretation of them through the two dichotomies are compared. For corroborating this interpretation, I add Leibniz's two labyrinths of the human reason.

| **Kant's two distinctions** | **Two basic dichotomies' interpretation** | **Leibniz's two labyrinths** |
|---|---|---|
| *Analytic*: "[…] nothing is added by the predicate to the concept of the subject" | Belonging to a theory founded on axioms (AO) | Law |
| *Synthetic*: "[…] adds to the concept of the subject a predicate not conceived as existing within it, and not extracting from it by any process of mere analysis" | Belonging to a theory based on empirical data, from which the theory induces a method for solving a basic problem (PO) | Freedom |
| *A priori*: is "[...] independent of experience and even of all impressions of the senses" | Related to the infinitesimal analysis (AI) | Actual infinity |
| *Empirical*: "has its sources in the experience" (also "a posteriori") | Related to the operative method of ruler and compass; more in general, related to constructive mathematics (PI)[15] | Potential infinity |

**Tab. 1.** Kant's categories for the judgements, the two basic dichotomies and Leibniz's two labyrinths

## 6. Interpretation of Kant's judgements by means of the representative principles of the MSTs

In order to compare more accurately Kant's judgements with MSTs I characterize a MST also by the first principle shared by all theories pertaining to it. Previous papers illustrated them: Principle of causality for the Newtonian MST, Principle of an extremant for the Lagrangian MST, Principle of existence of a mathematical being for the Cartesian MST, Principle of limitation for the Carnotian MST (Drago 2011b; Drago in press).

The analytic *a priori* judgement fits the principle of causality (as well as the metaphysical basic notions and principles of Newton's mechanics) of the Newtonian MST.[16] The synthetic a posteriori judgement corresponds to those propositions which are obtained by the empirical scientists when they are seen as incapable to achieve the theoretical heights of a scientific theory and even less the metaphysical heights of philosophy (Let us recall that for Kant the impossibility of a perpetual motion of the

[15] "Empirical" is the word employed also by both D'Alembert (1770-1775, p. 510) and Carnot (1803, Preface).
[16] See for ex. the analysis of the first principle by Hanson (1965) and also (Drago, Manno 1989; Drago1991).

Carnotian MST is not a judgement). This Kant's attitude corresponds to the long scientific tradition to see any limitation to the scientific research as an insurmountable bound for an arguing at the theoretical level.

Kant's "invention" of the synthetic *a priori* judgement corresponds to the formulations of theoretical physics each relying on an a priori extremant principle (AI) as applied to an operative, empirical framework (PO); i.e. Maupertuis' formulation based on the least action and Lagrange's formulation of the extremant of a particular form of the action. This relationship is in agreement with a recent interpretation of Kant's philosophy of the knowledge by the authoritative scholar Kitcher, who considered this philosophy as essentially a PO theory.[17] Hence, Kant's invention deserves the merit to have echoed inside the philosophy of knowledge the most impressive advancements of the "hard" science of his time.[18]

| **Kant's judgements** | **Analytic *a priori* (Metaphysics)** | **Synthetic *a priori* (Kant's invention)** | **Analytic *a posteriori* (Lacking in Kant)** | **Synthetic *a posteriori* (Empirism)** |
|---|---|---|---|---|
| **Four first principles in Theoretical Physics** | Cause-force Principle F=ma<br><br>Principle of causality | Extremants, Principle of least action, of least squares, etc.<br><br>Principle of an extremant | Huygens' principle<br><br>Principle of existence of a mathematical being | Impossibility of a perpetual motion, Principle of virtual velocities<br><br>Principle of limitation |
| **Two options and MSTs** | AO&AI Newtonian MST | PO&AI Lagrangian MST | AO&PI Cartesian MST | PO&PI Carnotian MST |

**Tab. 2.**Kant's basic notions, the two options, the MSTs and the first principles of physical theories

In correspondence to the first principle of the existence of a mathematical being there is no Kant's judgement. This fact is explained by three reasons. The representative physical theory of this MST, geometrical optics, was founded by Huygens on the

[17] (Kitcher 1983), in particular pp. 402-405 characterising the PO feature of Kant's theory.

[18] According to both Poincaré (1887) and the four MSTs (Drago 1999; Drago 2011a) there exist four main geometries. By comparing Kant's categories to this best instance of a structure variety in mathematics. One has to attribute the analytic a priori to the Euclidean geometry, owing to its metaphysical tradition of absolute certainty (and also the idealistic definitions of its basic notions); in opposition, the synthetic *a posteriori* has to be attributed to the hyperbolic geometry, that Lobachevsky built on merely empirical notions (Lobachevsky 1835-1838; Lobachevsky 1840); Kant's invention of the synthetic *a priori* judgement corresponds to Minkowsky geometry, since it is based on the velocity *c* considered as an extremant (also in comparison with the other geometries Minkowsky geometry – again the most advanced among the four issues – since it is based on the light speed as an asymptotic bound). The missed judgement – the analytic a posteriori – corresponds to the elliptic geometry. Although it was discovered in 1734 by the empiricist philosopher T. Reid, it has been ignored by the idealist Kant, and also by the mathematicians along a century and half.

principle of a wave as resulting from a mathematical operation of convolution of the wavelets. But at Kant's time it was at all dismissed since the prevailing foundation was Descartes's, whose basic notion was a light beam which is at all independent from the time, instead time in Kant's system plays the role of an a priori transcendental category of our knowledge. Moreover, at Kant's time geometrical optics was included as a particular case of Newton's mechanics.[19]

In sum, Kant's three kinds of judgements, although inadequate to represent theoretical physics since they missed the Cartesian MST and depreciated the Carnotian MST, implicitly gave philosophical relevance to the Lagrangian model (e.g. Lagrangian Mechanics). No surprise if subsequent philosophers, by suitably enlarging Kant's ideas and by means of analogical arguments resulted not inadequate with respect to the cultural matrix of the new theoretical advancements in science, which in general are obtained as mathematical development of Lagrangian functions.

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[19] A correspondence with Aristotle's four kinds of causes, suggests that Kant "invented" the efficient cause in a time when in the social life the leading notion of historical progress prevailed on the leading notion of tradition; and he missed the final cause, of course because this notion was inconceivable in Kant's metaphysical terms.

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