Newton's *Principia* Geneva edition: the action-and-reaction law. Historical and Nature of Science reflexions

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Abstract: The Geneva edition ([1739-1742] 1822) of Newton's *Principia* is a very treasure for the historians of physics and mathematics. For, the editors added a series of notes, which are longer than Newton's text itself. The explanations contained in such notes are important to grasp the way in which the spread of Newton's thought was realized in the continental Europe. Based on our previous studies, in this contribution we present a case study from Geneva edition adding Nature of Science educational reflexions.

Keywords: Newton, *Principia*, *Scholium*, action-reaction law, Geneva edition, infinitesimal of different orders.

1. Introduction

The Geneva edition of Newton's *Principia*[2] has unique features, which make this text a relevant document within the history of the reception and spread of Newton's physics in the cultural milieu of continental Europe around the forties of the XVIII century. The edition was not conceived only for specialists, rather it was thought of as a sort of encyclopaedia explaining all the aspects of Newton's mechanics to an expert but rather vast public.[3] We have faced the problems connected to the nature of the notes, to the personalities of the editors and to the general structure of this immense work. We are not going to repeat what we have already clarified. Rather, we prefer to add a small *tessera* to the mosaic of our explanations to the notes contained in the Geneva edition. This means to face a case study in order to provide the reader with an example how the editors worked. We have chosen a note to the famous *Scholium* inserted by Newton at the end of the celebrated section of his masterpiece entitled *Axioms or Laws of Motion* (*Axiomata sive Leges Motus*; NGE, pp. 15-44):

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[2] Hereafter we refer to Newton Geneva Edition (NGE, Public domain) in (Newton 1822, I). See also (Newton 1736; 1746a; 1729; 1746b; 1780-1785; 1972; 1803), (Wallis, Wallis 1977), (Pisano 2017).
[3] The realization of a larger project (Oxford University Press, 5 vols.) is expected by 2020. See, recently, (Bussotti, Pisano 2014a, b), (Pisano, Bussotti 2016a, b). See also (Pisano, Agassi, Drozdova 2017), (Pisano, Capecchi 2015), (Pisano, Fichant, Bussotti, Oliveira 2017), (Pisano 2015).

> Law I. Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon;
>
> Law II: The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.
>
> Law III: To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts [*Lex III. Actioni contrariam semper et æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales et in partes contrarias dirigi*] (Newton 1822, I, pp. 15-17).[4]

The note tells us something interesting as to the way, in which different order of infinitesimals were conceived in that period. We will briefly expound the content of Newton's *Scholium* and, afterwards, we are presenting the note we have chosen (NGE).

2. On the action-and-reaction law

The *Scholium*, which concludes the section entitled *Axioms or Laws of Motion*, is almost as long as the rest of the entire section. Newton presents scientific considerations to show his concept of force and his laws of motion. These allow him to face a series of problems, already addressed by other physicists, with ease and within a general perspective. Some interesting reasoning and mental experiment follow to present the validity of the action-and-reaction law. From a conceptual point of view, the *Scholium* can be divided into four parts:

1. In the first one, Newton shows that Galileo's results, according to which the free fall of the bodies is proportional to the square of the time and the trajectory of a projectile – neglected air resistance – is a parabola, can be explained taking into account the first two laws and their corollaries. For, Newton writes, the constant gravity produces equal velocities in the single time-intervals. This means the velocities are as the times and, hence, the spaces as the squares of the times. By means of this result and of the parallelogram rule – explained by Newton in the Corollary I – (NGE, pp. 17-19) it is then possible to realize why the motion of a projectile is a parabola. This aspect plays and important role in the Nature-of-Science, teaching physics, geometry-mathematics and are related to the geometrical models in physics. A possible alterative view was represented by the *principle of virtual works* (Pisano 2017) connected to engineering from XIX century. Still, nowadays the didactic on the subject depends on these two approaches.

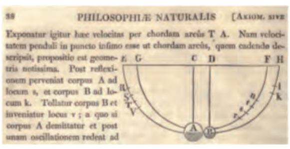
[4] See also (Newton 1803, I, pp. 19-20).

2. The second conceptual core of the *Scholium* regards the fact that the laws of the pendulum and the rules of the impact for the hard bodies can be deduced by the three laws and their corollaries. Newton points out that such rules were formulated by Wren, Wallis and Huygens, he defines the most illustrious geometers of his time (*aetatis superioris geometrarum facile principes*; NGE, p. 36), but they can be easily obtained thanks to his concept of force and his axioms. It would be interesting to add these Newtonian performances on *Axioms or Law of motion* at the higher education.
3. To prove the validity of action-and-reaction law and derive the fundamental law of *momentum conservation* (NGE, Corollary III, pp. 23-26). In his words:

> But to prevent an objection that may perhaps be alledged [alleged] against the rule, for the proof of which this experiment was made, as if this rule did suppose that the bodies were either absolutely hard, or at least perfectly elastic (whereas no such bodies are to be found in nature), I must add. That the experiments we have been describing, by no means depending upon that quality of hardness, do succeed as well in soft as in hard bodies. For if the rule is to be tried in bodies not perfectly hard, we are only to diminish the reflexion in such a certain proportion as the quantity of the elastic force requires. By the theory of Wren and Huygens, bodies absolutely hard return one from another with the same velocity with which they meet. But this may be affirmed with more certainty of bodies perfectly elastic. In bodies imperfectly elastic the velocity of the return is to be diminished together with the elastic force; because that force (except when the parts of bodies are bruised by their congress, or suffer some such extension as happens under the strokes of a hammer) is (as far as I can perceive) certain and determined, and makes the bodies to return one from the other with a relative velocity, which is in a given ratio to that relative velocity with which they met. This I tried in balls of wool, made up tightly, and strongly compressed. For, first, by letting go the pendulous bodies, and measuring their reflexion, I determined the quantity of their elastic force; and then, according to this force, estimated the reflexions that ought to happen in other cases of congress. And with this computation other experiments made afterwards did accordingly agree; the balls always receding one from the other with a relative velocity, which was to the relative velocity with which they met as about 5 to 9. Balls of steel returned with almost the same velocity: those of cork with a velocity something less; but in balls of glass the proportion was as about 15 to 16. And thus the third Law, so far as it regards percussions and reflexions, is proved by a theory exactly agreeing with experience (NGE, pp. 39-40; translation from Motte's edition).

Newton hypothesized two bodies *A* and *B* (Figure 1) suspended to the chords *AC* and *BD* respectively. He considered the circular pendulums *EAF* and *GBH* and supposed that – removed the body *B* – the body *A* starts from the point *R* and, because of the air resistance, it does not come back to *R* after an oscillation, but to *V*. The fourth part of *RV*, namely the arch *ST*, will hence express the retardation of the descent from *S* to *A*. This means that, if the body *A* starts from *S*, its velocity, when it reaches the point *A*, will be the same as if the body would fall in the void from *T*. Thence, the chord of the arch *TA* represents the velocity in *A*. By the same reasoning, Newton proves that the chord of the arch *tA* represents the velocity of *A* after one reflection. By a similar method, he determines the place *l*, which has, for the body *B*, the same meaning as the place *t* for the body *A*. In this manner, the product of the mass of *A* by the chord *TA* represents the quantity of motion of the body *A* before the reflection; the product of *A* by the chord *tA* that quantity after the reflection, and the product *B* by the chord *Bl* represents the quantity of motion of *B* immediately after the reflection. The result after the impact of the bodies *A* and *B* always confirms the validity of the action-and-reaction law and the *momentum conservation*. Newton varied the empirical hypotheses-conditions by modifying the bodies' masses, the lengths of the pendulums and the rigidity of the bodies. The action-and-reaction law was confirmed.

4. The fourth section of the *Scholium* is connected to the previous one. Newton proposes one of those simple, clear and ingenious reasoning, which characterize the whole of his production. The problem is to show the validity of the action-and-reaction law as to the attractive forces. Let us suppose, Newton writes, two bodies *A* and *B* are mutually attracted. Let us pose an obstacle between the two, so that the impact is avoided. If the body *A* were attracted by *B* more intensely than *B* by *A*, then the obstacle would be pressed by *A* more than by *B*. This implies that the system of the two bodies and the obstacle will proceed in the direction *A*→*B ad infinitum* and with an accelerated motion. However, this is impossible because of the first law (inertia), thence the two bodies *A* and *B* will press the obstacle with the same intensity and will mutually attract with the same force. This implies the validity of the action-and-reaction law. Newton claims to have proved the validity of this assertion by means of experiments carried out with a calamite and iron. He proves then that the parts of the Earth attract each other with the same intensity. In the final remarks of the *Scholium* he introduces, in practice, the concept of *work* as he claims the bodies, whose velocities are inversely proportional to their innate forces, are equivalent in the impact, in the reflection and in their capabilities to move mechanical instruments. Therefore, the concept of work is attributed to the action-and-reaction law.


38 PHILOSOPHIÆ NATURALIS [AXIOM. SIVE

Exponatur igitur haec velocitas per chordam arcûs T A. Nam velocitatem penduli in puncto infimo esse ut chordam arcûs, quem cadendo descripsit, propositio est geometris notissima. Post reflexionem perveniat corpus A ad locum s, et corpus B ad locum k. Tollatur corpus B et inveniatur locus v; a quo si corpus A demittatur et post unam oscillationem redeat ad

Fig. 1. The figure used by Newton to describe geometrically, and confirm physically, the action-and-reaction law (NGE, I, p. 38). Image: public domain

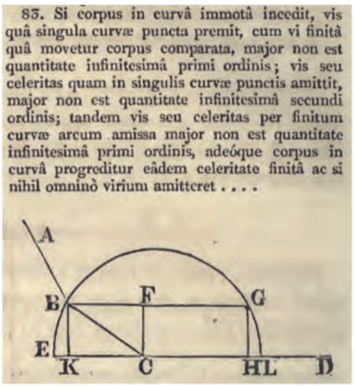

83. Si corpus in curvâ immotâ incedit, vis quâ singula curvæ puncta premit, cum vi finitâ quâ movetur corpus comparata, major non est quantitate infinitesimâ primi ordinis; vis seu celeritas quam in singulis curvæ punctis amittit, major non est quantitate infinitesimâ secundi ordinis; tandem vis seu celeritas per finitum curvæ arcum amissa major non est quantitate infinitesimâ primi ordinis, adeóque corpus in curvâ progreditur eâdem celeritate finitâ ac si nihil omninò virium amitteret . . . .

Fig. 2. The figure used by the editors to prove the theorem we analyse in the text (NGE, p. 34). Image: public domain

3. On the motion along a curve, from a Geneva edition's note

In the Geneva edition, the *Scholium* and the notes added by the editors include thirteen pages (NGE, pp. 32-44) and thirty-one notes, numbered from 75 to 105, in which several subjects connected to the concept of force and to the three Newton's laws are dealt with. The editors refer to a series of well-known problems as the descent along an inclined plane or the law of pendulum, but also to some less known aspects related to the properties of the forces. We will analyse the note 83 (NGE, p. 34), which is interesting from a physical and mathematical point of view. There is also an educative aspect because teaching physics by Nature-of-Science, taking into account the relationship between physics and mathematics, permits to go into the foundations of physics without losing mathematical description of a physical phenomenon.

The theorem proved by the editors is the following one: if one body moves on a curve, the force, with which the body presses the points of the curve, if compared with the finite force moving the body, is not bigger than a first order infinitesimal. The force or the velocity, which is lost in the single points of the curve, is not bigger than a second order infinitesimal quantity. Finally, the force or the velocity lost along a finite arch of the curve is not bigger than a first order infinitesimal quantity. Hence, the body proceeds along the curve with the same finite velocity as if it lost no force at all. The editors imagine the trajectory *ABCD* as a sort of material constrain (Figure 2).

The proof runs like this: the editors consider a curve to be a polygon *A*, *B*, *C*, *D*, etc. composed of innumerable and infinitesimals right sides *AB*, *BC*, *CD*,… Two of them, for example *BC* and *CD* comprehend an angle, which is less than two right angles only by an infinitesimal quantity, so that, prolonged the side *CD* in *E*, the external angle *BCE* is infinitesimal. Let us describe the semicircle *EBGL* with centre *C* and radius *CB*. From the point *B* be traced the perpendicular *BK* to the straight line *ED*. Let us complete the rectangle *KF*. The motion of the body along the side *BC* can be decomposed into the two motions along *BK* and *BF* or *KC* (Corollary I by Newton, the parallelogram rule; NGE, pp. 17-19). This granted, it is evident that the force or the velocity, with which the body moves in the side *CD* and presses or hits that side is represented by the perpendicular *FC* or *BK*. The velocity after the impact (if the body has no elasticity) is indicated by the straight line *KC* or *CH*. The velocity lost as a consequence of the impact in *C* is indicated by the straight line *EK*, because *EK* is the difference of the lines *BC* and *KC*, that is the difference of the velocities before and after the impact. If the angle *BCK* were finite, the straight line *BK* would have a finite ratio to the lines *BC* and *KC*. While decreasing the angle *BCK*, this ratio decreases continuously and hence it becomes infinitesimal when the angle *BCK* is infinitesimal. Therefore, *BK* or the force, with which the body presses the curve in *C*, is not greater than a first order infinitesimal quantity. Actually, since in the circle $EK:BK=BK:KL$, then *EK* will be an infinitesimal quantity in respect to *BK*. Thus, relying upon what proved, *BK* is infinitesimal in respect to *BC*, or *KC* and, hence, in respect to *KL*. Therefore, the velocity or the force lost in a point *C* is not bigger than a second order infinitesimal quantity. Since the velocity the body loses in the single sides of the trajectory *AB*, *BC*, *CD* is not bigger than a second order infinitesimal, then the body, while moving along the sides of the curve, whose number is infinite, namely, while moving along a finite arch of the curve, cannot lose a velocity greater than a first order infinitesimal quantity, which is the sum of second order infinitesimal quantities. Without taking into account such a quantity, the body continues its motion along the curve as if it lost no force, which proves the theorem completely.

4. Concluding remarks

The above examples and related discussion are interesting because the Geneva edition explicitly shows how the concept of infinitesimal was used at that time within physics. In that context, the idea of considering a curve as a polygon of infinite sides is also expressed. This is also connected with the different types of infinitesimal quantities used by Newton and the Newtonians.[5]

We have presented this case study for the reader to get an idea of the notes added by the editors of the Geneva edition to Newton's text. There are several notes, which are a precise – not often, but sometimes pedantic – specification and explanation, but there are also notes – as the one we have considered – that analyse many specific cases and circumstances, not directly faced by Newton, nonetheless connected to his physics. These notes are the most interesting because they represent a clear picture of two aspects:

1. The numerous applications and specifications to which Newton's physics can lead. Newton himself and the most important physicists after him did not develop all the single details of physics, because the advanced research

[5] We do not have room to deal with such a fascinating subject, thence we refer to our recent publication: (Pisano, Bussotti 2017).

demanded new conceptually important results. Thence, details, which might be defined didactical applications of Newton's mechanics, did not receive much attention. However, since Geneva edition has also the aim to popularize Newton's thought, these details are developed, which is amazingly interesting to fully realize how wide the perspective of Newton's physics is.

2. There are several notes, in which the editors clarify the development of physics after Newton's work. Thence, they refer to the works, results and methods of the main physicists, who operated after Newton. In this manner, a synoptic picture of the whole mechanics developed until the forties of the XVIII century is explained.

Because of this, such edition is worth of the attention of the scientists, historians and philosophers of science. The Geneva edition also incorporates, nowadays, important educational implications of Nature of Science, its history, philosophy and epistemology, for teaching physics and mathematics. The significance of models and modelling for science education is also connected to the use of metaphors, analogy, visualization, simulations and animations in science.

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