The classical ether-drift experiments
An enigma for physics and history of science

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Abstract: Motivated by general theoretical arguments, the classical ether-drift experiments (Michelson-Morley, Illingworth, Miller, Joos, …) were recently re-considered from scratch in Consoli *et al.* (2013). The conclusion of that analysis is that their standard null interpretation is far from obvious. In fact, by using Lorentz transformations to connect the Earth's frame to a hypothetical preferred frame, the small observed effects point to an average Earth's velocity of about 300 km/s, as for most cosmic motions. A common feature is the irregular behaviour of the data. While this has motivated, so far, their standard interpretation as instrumental artefacts, the new re-analysis of the very accurate Joos experiment gives definite indications for that type of Earth's motion associated with the Cosmic Microwave Background anisotropy and thus leaves little space for this traditional interpretation. The new explanation requires instead a view of the vacuum as a stochastic medium, similar to a fluid in a turbulent state of motion, in agreement with basic foundational aspects of both quantum physics and relativity. The overall consistency of this picture with the present experiments with vacuum optical resonators and the need for a new generation of dedicated ether-drift experiments are also emphasized. Without this definite clarification, the classical ether-drift experiments represent an enigma for physics and history of science.

Keywords: Ether-drift experiments, Stochastic vacuum, Quantum physics, Relativity.

1. Introduction

The Michelson-Morley experiment (Michelson, Morley 1887) has represented a fundamental breakthrough in the history of physics. Indeed, it was its result, too small to meet any classical prediction, to stimulate the first formulations of the relativistic effects and later on to induce Lorentz (1904), and Poincaré (1905), to derive a particular set of transformations of the space-time coordinates (Lorentz Transformations): "Applying one of such transformations amounts to apply an overall translation to the whole system. Then two frames, one at rest in the ether and one in uniform translation, become the perfect images of each other". This statement of Poincaré in 1905 was the precise formalization of the Principle of Relativity, already proposed by him (Giannetto 1993) in *La Science et l'Hypothèse* (Poincaré 1902) and at the 1904 St. Louis Conference. This first historical phase and its relation with Special Relativity (Einstein 1905) is well described in an Einstein's interview delivered in 1955, a few months before his death. When asked, once more, about his original view and the relation with previous work he said:

> There is no doubt that the special theory of relativity, if we regard its development in retrospect, was ripe for discovery in 1905. Lorentz had already observed that for the analysis of Maxwell's equations the transformations which later were known by his name are essential, and Poincaré had even penetrated deeper into these connections. Concerning myself, I knew only Lorentz's important work of 1895 (the two papers quoted above in the German text) but not Lorentz's later work, nor the consecutive investigations by Poincaré. In this sense my work of 1905 was independent. The new feature of it was the realization of the fact that the bearing of Lorentz-transformations transcended its connection with Maxwell's equations and was concerned with the nature of space and time in general. A further new result was that the "Lorentz invariance" is a general condition for any physical theory (Born 1956a, pp. 248-249, Born 1956b, p. 194).

This premise is essential to properly frame the Michelson-Morley experiment in the evolution of scientific thought. At the same time, nowadays, there is the tendency to consider this experiment, and its repetitions (Illingworth 1927; Joos 1930; Miller 1933) as *old stuff* for which there is nothing more to refine or clarify. Instead, as we are going to illustrate, there is still a lot to be clarified about these classical *ether-drift* experiments, hopefully through cooperation between physicists and historians of science. To introduce this matter, let us start from scratch by first observing that, in spite of the deep conceptual difference, Einstein's view of relativity and the Lorentzian perspective are equivalent with respect to most experiments where one just compares the relative measurements of two observers. This conclusion was, for instance, already clearly expressed by Ehrenfest (1913) in his lecture *Zur Krise der Lichtäther-Hypothese*[1] as follows:

> So, we see that the ether-less theory of Einstein demands exactly the same here as the ether theory of Lorentz. It is, in fact, because of this circumstance, that according to Einsteinian theory an observer must observe exactly the same contractions, changes of rate, etc. in the measuring rods, clocks, etc. moving with respect to him as in the Lorentzian theory. And let it be said here right away and in all generality. As a matter of principle, there is no experimentum crucis between the two theories. (Ehrenfest 1913, pp. 17-18, quoted in Dorling 1968, p. 69).

One can easily understand this because, independently of all interpretative aspects, the basic quantitative ingredients, namely Lorentz transformations, are the same in both formulations. Their validity will be assumed in the following to discuss the possible existence of a preferred reference frame.

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[1] *Zur Krise der Lichtäther-Hypothese (On the crisis of the light ether hypothesis)*, lecture given by Ehrenfest in Leyden on December 4, 1912.

The substantial equivalence then reflects the fundamental property of the Lorentz group for which two observers **S'** and **S''**, individually related to the preferred frame by Lorentz transformations with velocities **v'** and **v''**, are connected by a Lorentz transformation with relative velocity fixed by the relativistic composition rule (for simplicity we restrict to one-dimensional motion)

$$\mathbf{v}_{\mathrm{rel}} = \frac{\mathbf{v'} - \mathbf{v''}}{1 - \frac{\mathbf{v'}\mathbf{v''}}{\mathbf{c}^2}} \quad . \tag{1}$$

Then, one may get the impression that the present supremacy of Einstein's interpretation is precisely due to the null results of the ether-drift experiments where one attempts to measure an absolute velocity. But, again, this is wrong. In a Lorentzian view, if the velocity of light $\mathbf{c}_\gamma$ propagating in the various interferometers coincides with the basic parameter **c** entering Lorentz transformations, relativistic effects conspire to make undetectable the individual velocity of each observer. Therefore, a null result of the ether-drift experiments should *not* be automatically interpreted as a confirmation of Special Relativity. As stressed by Ehrenfest, the motion with respect to the preferred frame might remain unobservable, yet one could interpret relativity *à la* Lorentz. As emphasized by Bell (1987), this could be crucial, for instance, to reconcile faster-than-light signals with causality and thus provide a very different view of the apparent non-local aspects of the quantum theory. This discussion should convince that Einstein's axiomatic interpretation, which is the one presented today in most textbooks, might obscure fundamental aspects of nature. After this basic premise, let us look more closely at the ether-drift experiments. Here, the curiosity of a reader should be stimulated by the claims of greatest experts, e.g. Hicks (1902) and Miller (1933), which, over the years, have stressed the importance of small residual effects by seriously questioning the standard null interpretation. Additional interest may derive by exploring those approaches where Lorentz symmetry, rather than being postulated from scratch, represents an *emergent* phenomenon. In this modified perspective, one could conceivably detect the effects of absolute motion thus giving support to the idea of a non-zero ether-drift effect. Finally, other motivations may derive from those modern representations of the particle physics vacuum as a condensate of elementary quanta. These condense because their trivially empty vacuum is a meta-stable state and not the true ground state of the theory. This situation can be summarized by saying that:

> What we experience as empty space is nothing but the configuration of the Higgs field that has the lowest possible energy. If we move from field jargon to particle jargon, this means that empty space is actually filled with Higgs particles. They have 'Bose condensed' ('t Hooft 1997, p. 70).

The explicit translation from field jargon to particle jargon, with the substantial equivalence between the effective potential of quantum field theory and the energy density of a dilute particle condensate, can be found for instance in Consoli, Stevenson (2000). The symmetric vacuum, with vanishing Higgs field, will eventually be re-established by heating the system above a critical temperature where the condensate "evaporates". This temperature in the Standard Model is so high that one tends to approximate the ordinary vacuum as a zero-temperature system. In this limit, the physical vacuum should behave as a form of superfluid medium (Volovik 2001) that is not trivially empty but through which bodies should flow without any apparent friction.

Clearly, this form of quantum vacuum is not the kind of ether imagined by Lorentz. However, if possible, this modern view of the vacuum state is even more different from the empty space-time of Special Relativity that Einstein had in mind in 1905. Therefore, one may ask if Bose condensation, i.e. the macroscopic occupation of the same quantum state, say $\mathbf{k} = 0$, in some reference frame can represent the operative construction of a *quantum ether*. This characterizes the physically realized form of relativity and could play the role of preferred reference frame in a modern Lorentzian approach. The key point is that exact Lorentz invariance of the vacuum state requires imposing the problematic condition of a zero vacuum energy (see e.g. Streater, Wightman 1964). This only holds in theories with an exact supersymmetry, which are not phenomenologically viable. Alternatively, with a non-zero energy of the reference vacuum state, a Lorentz transformation could produce an energy-momentum flow in a moving frame $\mathbf{S'}$. This would behave as an effective thermal gradient and could affect the various forms of matter differently, for instance by producing tiny convective currents in loosely bound systems such as gases or dissipating mainly by heat conduction in strongly bound systems such solids or liquid dielectrics. In the first case, convective currents of the gas molecules along the light path could give rise to a slight anisotropy of the velocity of light that, in principle, could explain the small residuals of the classical experiments.

With this in mind, we have undertaken a careful reanalysis of these classical experiments to first check Hicks' and Miller's claims and then compare with quantitative predictions in a modern theoretical framework. The results of our analysis have been presented in Consoli *et al.* (2013), to which we address the reader for all details. The conclusions of that work are striking: Hicks and Miller were right. The traditional null interpretation is far from obvious. In addition, by using Lorentz transformations to connect the Earth's frame to a hypothetical Lorentzian preferred frame, the small observed residuals point to an average Earth's velocity of about 300 km/s, as in most cosmic motions. A common feature is the irregular behavior of the data. While this has motivated, so far, their standard interpretation as instrumental artifacts, our new re-analysis of the very accurate Joos experiment gives definite indications for the type of Earth's motion associated with the CMB anisotropy and thus leaves little space for this traditional interpretation. The new explanation requires instead a view of the vacuum as a stochastic medium, similar to a fluid in a turbulent state of motion, in agreement with basic foundational aspects of both quantum physics and relativity. The overall consistency of this picture with the present experiments with vacuum optical resonators and the need for a new generation of dedicated ether-drift experiments were also emphasized in Consoli *et al.* (2013).

In the following, we will summarize these results, by first discussing in Sects. 2 and 3 the basics of the ether-drift experiments and pointing out some non-trivial aspects, related to the nature of the vacuum as a quantum medium, which have always been overlooked in the standard analysis. Then, in Sect. 4, as an example, we will review the original experiment performed by Michelson and Morley in 1887. A summary of the other experiments, as presented in Consoli *et al.* (2013), and in particular the intriguing aspects of the most precise version performed by Joos in 1930, can be found in the final Sect. 5.

2. Basics of the ether-drift experiments

Traditionally, the Michelson-Morley experiment is presented from the point of view of an observer sitting in the reference frame of the ether, say **Σ**. However, the experiment is performed in the Earth's frame, say **S'**. One only needs very little thought to realize that everything becomes much simpler by adopting the point of view of this observer. In fact, by assuming the validity of Lorentz transformations, the length of a rod, in the frame where this is at rest, does *not* depend on its orientation. Therefore any non-zero fringe shift, upon rotation of a Michelson interferometer, can only be due to an *anisotropy* of the velocity of light, say $\mathbf{c_\gamma\ (\theta)}$, where $\boldsymbol{\theta}$ indicates the angle between the direction of light propagation and the Earth's velocity. The interference fringes then depend on the time difference for light propagating back and forth along a path of length **D** over directions $\boldsymbol{\theta}$ and $\boldsymbol{(\pi/2 + \theta)}$, which is given by the expression (see Fig. 1):

$$\Delta T(\theta) = \frac{2D}{\bar{c}_\gamma(\theta)} - \frac{2D}{\bar{c}_\gamma(\pi/2+\theta)}\ . \tag{2}$$

where we have introduced the *two-way* velocity (the only one which can be unambiguously measured)

$$\bar{c}_\gamma(\theta) = \frac{2c_\gamma(\theta)c_\gamma(\pi+\theta)}{c_\gamma(\theta)+c_\gamma(\pi+\theta)}\ . \tag{3}$$

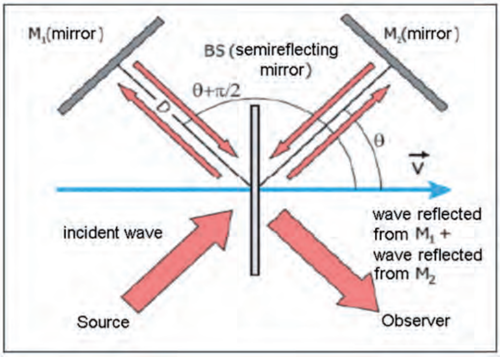


Fig. 1. The typical scheme of Michelson interferometer

To analyze this quantity, let us start from the standard assumption to approach ether-drift experiments: any possible light anisotropy should vanish when both the observer and (the container of) the medium where light propagates are at rest in the hypothetical **Σ**. Therefore, in the physical case where instead both the observer and (the container of) the medium are at rest in the laboratory frame, any possible anisotropy should vanish identically in the *two* limits when either **V** = 0 (i.e. when **S'** ≡ **Σ**) or when the velocity of light coincides with the basic parameter **c** entering Lorentz transformations. The classical ether-drift experiments were performed in gaseous media (air or helium at atmospheric pressure) where the refractive index $\mathbf{N = 1+\varepsilon}$ is extremely close to unity. Thus, by expanding in the two small parameters $\boldsymbol{\beta} = \mathbf{V/c}$ and $\boldsymbol{\varepsilon}$, any possible anisotropy should start to $\mathbf{O(\varepsilon\,\beta)}$ for the one-way velocity and to $\mathbf{O(\varepsilon\,\beta^2)}$ for the two-way velocity which is invariant under the replacement $\boldsymbol{\beta} \to -\boldsymbol{\beta}$. At the same time, for any fixed $\boldsymbol{\beta}$, the two-way is invariant under the replacement $\boldsymbol{\theta} \to \boldsymbol{\pi} + \boldsymbol{\theta}$. Therefore, one can write down the general expression

$$\bar{c}_\gamma(\theta) = \frac{c}{N}\left[1 - \varepsilon\beta^2 \sum_{n=0}^{\infty} \varsigma_{2n} P_{2n}(\cos\theta)\right]. \tag{4}$$

where the angular symmetry of the two-way velocity has been expressed as an infinite expansion of even-order Legendre polynomials with unknown coefficients $\boldsymbol{\varsigma_{2n}}$. In Einstein's Special Relativity these coefficients should vanish identically. In a Lorenzian formulation, on the other hand, there is no reason why they should vanish *a priori*.

A crucial point is that *exactly* the same equation (4) is obtained (Consoli *et al.* 2013) if, along the light path, there were convective currents of the gas molecules associated with an Earth's absolute velocity **V**. This alternative derivation, motivated by the idea of the fundamental energy-momentum flow that might be associated with a non-zero vacuum energy, provides a dynamical basis for Eq. (4) and a new approach to the unexplained residuals of the classical ether-drift experiments where light was still propagating in gaseous systems. In this scheme, one can also understand the difference with experiments performed in strongly bound systems, such as solid or liquid transparent media, as in the Shamir-Fox experiment (Shamir, Fox 1969). Being aware that the classical experiments might also admit a non-null interpretation proportional to $\mathbf{(N\text{-}1)\,\beta^2}$, they selected a medium where the effect of the refractive index could have been enhanced (i.e. perspex where **N** = 1.5). Since this enhancement was not observed, they concluded that the experimental basis of Special Relativity was strengthened. However, with the proposed mechanism, in solid and liquid dielectrics one expects that a small energy flux, generated by the motion with respect to the vacuum condensate, should mainly dissipate by heat conduction with no appreciable particle flow and no light anisotropy in the rest frame of the medium. Thus one has a physical argument to understand the two different behaviors.

Now Eq. (4) is exact, to the given accuracy, and could be used directly to compare with experiments by leaving out the first few $\boldsymbol{\varsigma_{2n}}$s as free parameters in the fits to the data. This general structure can however be compared with the particular form which is obtained by using Lorentz transformations (Consoli *et al.* 2013, appendix A) to relate the effective metric $\mathbf{g_{\mu\nu}}$, which governs light propagation when the gaseous medium is at rest in the laboratory **S'** frame to the reference, isotropic metric

$$\gamma^{\mu\nu} \equiv (N^2, -1, -1, -1) \tag{5}$$

which describes light propagation when the same gaseous medium is at rest in **Σ**. In this way, one finds

$$\bar{c}_\gamma(\theta) = \frac{c}{N}\left[1 - \varepsilon\beta^2(2 - \sin^2\theta)\right] \tag{6}$$

which corresponds to set in Eq. (4) $\varsigma_0 = 4/3$, $\varsigma_2 = 2/3$ and the remaining $\varsigma_{2n} = 0$ for all $\mathbf{n} > \mathbf{1}$. Eq. (6) represents a definite realization of the general structure in (4) and provides a partial answer to the problems posed by our limited knowledge of the electromagnetic properties of gaseous systems. As such, it can be adopted as a model for the two-way velocity of light. In this way, one obtains the anisotropy

$$\Delta\bar{c}_\theta \equiv \frac{\bar{c}_\gamma(\pi/2 + \theta) - \bar{c}_\gamma(\theta)}{c} \approx (N - 1)\frac{v^2}{c^2}\cos 2(\theta - \theta_0) \tag{7}$$

where the pair $(\mathbf{v}, \theta_0)$ describes the projection of **V** onto the relevant plane. By using Eq. (2), one then obtains the pattern of the interference fringes

$$\frac{\Delta\lambda(\theta)}{\lambda} = \frac{c\Delta T(\theta)}{\lambda} \approx \frac{D}{\lambda}\frac{v^2_{obs}}{c^2}\cos 2(\theta - \theta_0) \equiv A_2 \cos 2(\theta - \theta_0) \tag{8}$$

where we have introduced the amplitude of the 2nd-harmonic effect $\mathbf{A_2}$ and the *observable* velocity

$$v^2_{obs} \approx 2(N-1)v^2 \ll v^2. \tag{9}$$

In this way, in terms of the observable velocity, Eq. (8) has the same form as in the classical formula

$$\left[\frac{\Delta\lambda(\theta)}{\lambda}\right]_{class} = \frac{D}{\lambda}\frac{v^2}{c^2}\cos 2(\theta - \theta_0) \equiv A_2^{class}\cos 2(\theta - \theta_0). \tag{10}$$

Therefore, in this scheme, the interpretation of the experiments is transparent. According to Special Relativity, there can be no fringe shift upon rotation of the interferometer. In fact, the frame of isotropic propagation is always assumed to coincide with the laboratory frame **S'**, where (the container of) the gaseous medium is at rest, and thus one has $\mathbf{v_{obs}} = \mathbf{v} = \mathbf{0}$. On the other hand, if there were fringe shifts, one could try to deduce the existence of a preferred frame provided the following minimal requirements are fulfilled:

1. the fringe shifts exhibit an angular dependence of the type in Eq. (8),
2. by using gaseous media with different refractive index one gets consistency with Eq. (9) in such a way that different observable velocities correspond to the same kinematical **v**.

3. Reconsidering the classical ether-drift experiments

Classical ether-drift experiments were performed before relativity and quantum theory could be fully exploited. Thus, the model to compare with the data was the old classical prediction Eq. (10). In this scheme the predictions, although being formally $\mathbf{O(v^2/c^2)}$ were *large* as compared to the extraordinary sensitivity of the Michelson interferometer. One was also expecting smooth modulations because the only time dependence was associated with slow effects such as the daily Earth's rotation and its annual orbital revolution. As it is well known, this simple theoretical framework did *not* fit with the observations. In fact, the experimental fringe shifts, even though slightly larger than the experimental resolution, were much smaller than those classically expected for a velocity $\mathbf{v}$ = 30 km/s (the Earth's orbital velocity about the Sun and consequently, at that time, the minimum anticipated drift velocity). Also the observed pattern was highly irregular because observations performed at the same time on consecutive days could differ sizeably. This has always represented a strong argument to interpret the data as pure instrumental effects or "null results". As anticipated, however, Hicks (1902) and Miller (1933) have seriously questioned the traditional null interpretation. This suggests that, in some alternative framework, the small and irregular effects could acquire a definite physical meaning. To this end, let us consider the basic Eqs. (8) and (9). The main point of such representation is that the effects are now suppressed by the very small factor $2(\mathbf{N} - 1)$. Therefore, for air, where the refractive index $\mathbf{N}$ = 1.00029, the fringe shifts for the typical kinematical $\mathbf{v}$ = 300 km/s of most cosmic motions would be about 17 times smaller than those classically expected for $\mathbf{v}$ = 30 km/s from Eq. (10). For gaseous helium, where $\mathbf{N}$ = 1.000035, the effect would be even 140 times smaller.

Then, the real question might not concern the existence of the ether-drift effect itself but, rather, the theoretical model to compare observations performed at different times and in different places. Within the scheme of Eqs. (8) and (9), the crucial information is contained in the two time-dependent functions $\mathbf{v = v(t)}$ and $\boldsymbol{\theta_0 = \theta_0(t)}$, respectively the magnitude of the ether-drift effect and its apparent direction in the plane of the interferometer. So far, they have always been computed from of a cosmic Earth's velocity (with well-defined magnitude $\mathbf{V}$, right ascension $\boldsymbol{\alpha}$ and angular declination $\boldsymbol{\gamma}$ by using spherical trigonometry. Here, however, there is a logical gap. In fact, by comparing the Earth's motion with that of a body in a fluid, this standard picture amounts to a pure laminar flow where global and local velocity fields coincide. But the relation between the macroscopic Earth's motions and the ether-drift experiments depends on the physical nature of the vacuum. If this is a true quantum medium, the fringe shifts will likely exhibit the typical irregular (non-deterministic) pattern which characterizes any quantum measurement. Then, consistently with basic foundational aspects of quantum physics and relativity (for a review see e.g. Consoli *et al.* 2014), rather than adopting the simple classical model of a laminar flow, one could try to compare with models of *turbulent* flow, see Fig.2.

In this new perspective, the situation changes completely. In fact, in numerical simulations of highly turbulent flows at small-scales, see e.g. Fung *et al.* (1992), the parameters that describe the macroscopic motion of a body (the Earth's motion in our case) are only used to fix the limiting boundaries for a microscopic velocity field, which has instead an intrinsic *stochastic* nature. Therefore if, in agreement with Kolmogorov's theory of a fluid with vanishingly small viscosity (Kolmogorov 1941), the microscopic flow were assumed to be statistically isotropic, the stochastic velocity components would vary within the same range in all directions. In this case, the averages of all vectorial observables would vanish by simply increasing the number of observations and, in our case of the ether-drift experiments, by performing more and more observations, at the same sidereal time; fringe shifts would unavoidably average to zero. Clearly, such vanishing of the averages would *not* mean that there is no ether-drift but only that it becomes non trivial to reconstruct the kinematical parameters from microscopic measurements of the velocity of light in a laboratory. With this proviso, we shall now review the original 1887 Michelson-Morley experiment.

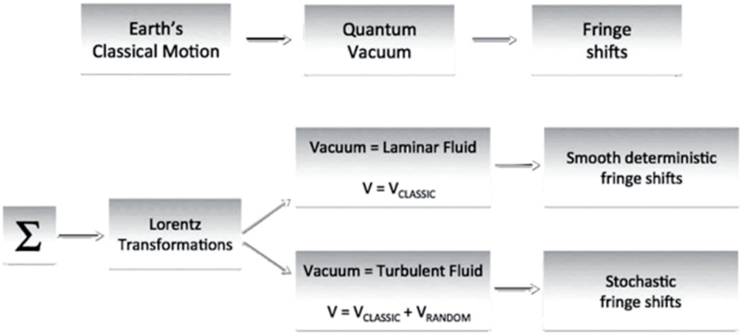


Fig. 2. The two possible ways to relate Earth's classical motion and fringe shifts

4. The original Michelson-Morley experiment

The Michelson-Morley experiment (Michelson, Morley 1887) is probably the most famous experiment in the history of physics. Its result and its interpretation have been (and are still) the subject of endless controversies. For instance, for some time there was the idea that, by taking into account the reflection from a moving mirror and other effects, the predicted shifts would be largely reduced and become unobservable. These points of view are summarized in Hedrick's contribution to the Conference on the Michelson-Morley experiment (Michelson *et al.* 1928), held in Pasadena in February 1927, which was attended by the greatest experts of the time, in particular Lorentz and Michelson. The arguments presented by Hedrick were, however, refuted by Kennedy (1935) in a paper of 1935 where, by using Huygens principle, he re-obtained to order $\mathbf{O(v^2/c^2)}$ the classical result of Eq. (10). In this way, the fringe shift is a second-harmonic effect, i.e. periodic in the angular range $\mathbf{[0, \pi]}$ whose amplitude $\mathbf{A_2}$ is predicted differently by using the classical formulas or Lorentz transformations (8) and (9). Now, for the Michelson-Morley interferometer the whole effective optical path was about D=11 meters, or $\mathbf{2 \cdot 10^7}$ in units of light wavelengths, so for a velocity $v = 30$ km/s the expected classical 2nd-harmonic amplitude was $\mathbf{A_2} \approx 0.2$ This value can thus be used as a reference point to obtain an observable velocity, in the plane of the interferometer, from the actual measured value of $\mathbf{A_2}$ through the relation

$$\mathbf{v_{obs} \approx 30km/s\sqrt{\frac{A_2}{0.2}}} \qquad (11)$$

Michelson and Morley performed their six observations in 1887, on July 8th, 9th, 11th and 12th, at noon and in the evening, in the basement of the Case Western University of Cleveland. Each experimental session consisted of six turns of the interferometer performed in about 36 minutes. As well summarized by Miller (1933), “the brief series of observations was sufficient to show clearly that the effect did not have the anticipated magnitude. However, and this fact must be emphasized, the indicated effect was not zero”. The same conclusion had already been obtained by Hicks (1902): “the data published by Michelson and Morley, instead of giving a null result, show distinct evidence for an effect of the kind to be expected”.

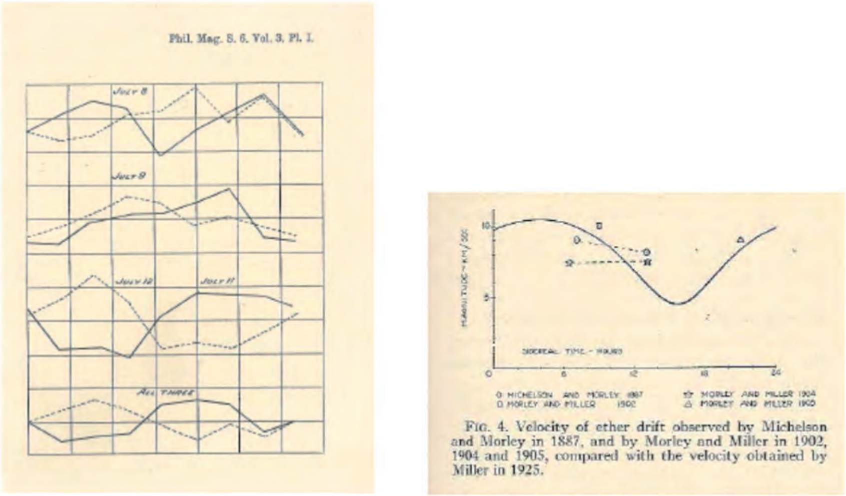


FIG. 4. Velocity of ether drift observed by Michelson and Morley in 1887, and by Morley and Miller in 1902, 1904 and 1905, compared with the velocity obtained by Miller in 1925.

Figs. 3-4. The Michelson-Morley fringe shifts as reported by Hicks. Solid and dashed lines refer respectively to noon and evening observation (left). The magnitude of the observable velocity measured in various experiments as reported by Miller (right)

Namely, there was a second-harmonic effect. But its amplitude was substantially smaller than the classical expectation, see Fig. 3. Quantitatively, the situation can be summarized in Fig. 4, taken from Miller (1933), where the values of the effective velocity measured in various ether-drift experiments are reported and compared with a smooth curve fitted by Miller to his own results as function of the sidereal time. For the Michelson-Morley experiment, the average observable velocity reported by Miller is about 8.4 km/s. Comparing with the classical prediction for a velocity of 30 km/s, this means an experimental 2nd-harmonic amplitude $\mathbf{A_2} \approx 0.016$ which is about twelve times smaller than the expected result. Now, neither Hicks nor Miller reported an estimate of the error on the 2nd-harmonic extracted from the Michelson-Morley data. To understand the precision of their readings, we can look at the original paper (Michelson, Morley

1887) where one finds the following statement: “The readings are divisions of the screw-heads. The width of the fringes varied from 40 to 60 divisions, the mean value being near 50, so that one division means 0.02 wavelength”.

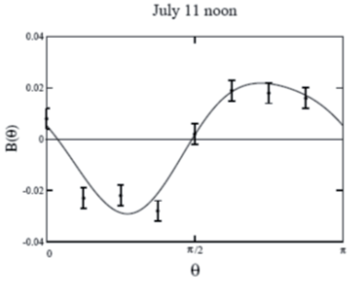


Fig. 5. A fit to the even combination B(θ) Eq. (12). The second harmonic amplitude is A_2^{EXP}=0.025±0.005 and the fourth harmonic is A_4^{EXP}=0.004 ± 0.005. The figure is taken from Consoli, Costanzo (2004a). Compare the data with the solid curve of July 11th shown in Fig. 3

Now, in their tables Michelson and Morley reported the readings with an accuracy of 1/10 of a division (example 44.7, 44.0, 43.5, …). This means that the nominal accuracy of the readings was ± 0.002 wavelengths. In fact, in units of wavelengths, they reported values such as 0.862, 0.832, 0.824, … Furthermore, this estimate of the error agrees well with Born’s book (Born 1962) where, when discussing the classically expected fractional fringe shift upon rotation of the apparatus by 90 degrees, about 0.37, he explicitly says: “Michelson was certain that the one-hundredth part of this displacement would still be observable” (i.e. 0.0037). Therefore, to be consistent with both the original Michelson-Morley article and Born’s quotation of Michelson’s thought, the value ± 0.004 was adopted (Consoli, Costanzo 2004a; Consoli *et al.* 2013) as an estimate of the error in the fits to the Michelson-Morley experimental data.

The values of all experimental fringes can be found in Consoli, Costanzo (2004a) and in Consoli *et al.* (2013). Notice only that, to meaningfully fit $\mathbf{A_2}$, one should first combine the data

$$\mathbf{B(\theta)=\frac{\Delta\lambda(\theta)+\Delta\lambda(\pi+\theta)}{2\lambda}} \tag{12}$$

to get rid of any odd harmonics, see e.g. Fig. 5. From our analysis, as one can see by inspection of Tab. 1, Miller’s average value $\mathbf{A_2} \approx 0.016$ is completely confirmed.

By using our Eqs. (8) and (9), these values give an observable velocity $\mathbf{v_{obs}} \approx$ **(8.4 ± 1.6)** km/s and, by taking into account the refractive index of air **N**=1.00029.., a true kinematical velocity **v** ≈ **(349 ± 66)** km/s.

While the individual values of $\mathbf{A_2}$ show a reasonable consistency, there are substantial changes in the apparent direction $\mathbf{\theta_0 = \theta_0(t)}$ of the ether-drift effect in the plane of the interferometer, the *azimuth*. This is the reason for the strong cancelations obtained when fitting together all noon sessions or all evening sessions. According to the usual interpretation, the large spread of the azimuths is taken as indication that any non-zero fringe shift is due to pure instrumental effects. However, as emphasized in Sect. 3, this type of discrepancy could also indicate an unconventional form of ether-drift where there are substantial deviations from the standard picture of the ether-drift as a smooth, periodic phenomenon. In any case, the observed strong variations of the azimuth are in qualitative agreement with the analogous values reported by Miller. To this end, compare with Fig. 22 of Miller (1933) and in particular with the large scatter of the data taken around August 1st, as this represents the epoch of the year which is closer to the period of July when the Michelson-Morley observations were actually performed. Thus one could also conclude that individual experimental sessions indicate a definite non-zero ether-drift but the azimuth does not exhibit the smooth trend expected from the conventional picture.

SESSION	A_2
July 8 (noon)	0.010 ± 0.005
July 9 (noon)	0.015 ± 0.005
July 11 (noon)	0.025 ± 0.005
July 8 (evening)	0.014 ± 0.005
July 9 (evening)	0.011 ± 0.005
July 12 (evening)	0.024 ± 0.005

Tab. 1. The amplitude of the fitted second-harmonic component A_2 for the six experimental sessions of the Michelson-Morley experiment

For completeness, we add that the large spread of the various $\boldsymbol{\theta_0}$ values might also reflect a particular systematic effect pointed out by Hicks (1902). As described by Miller (1933)

> before beginning observations the end mirror on the telescope arm is very carefully adjusted to secure vertical fringes of suitable width. There are two adjustments of the angle of this mirror which will give fringes of the same width but which produces opposite displacements of the fringes for the same change in one of the light-paths (Miller 1933, p. 210).

Since the relevant shifts are extremely small,

> the adjustments of the mirrors can easily change from one type to the other on consecutive days. It follows that averaging the results of different days in the usual manner is not allowable unless the types are all the same. If this is not attended to, the average displacement may be expected to come out zero, at least if a large number are averaged (Hicks 1902, p. 34).

Therefore averaging directly the fringe shifts from various sessions represents a delicate issue and can introduce uncontrolled errors. Clearly, this relative sign does not affect the values of $\mathbf{A_2}$ and this is why averaging the 2nd-harmonic amplitudes is a safer procedure.

However, it can introduce spurious changes in the apparent direction of the ether-drift. In fact, an overall change of sign of the fringe shifts is equivalent to replacing $\boldsymbol{\theta_0} \rightarrow \boldsymbol{\theta_0} + \boldsymbol{\pi/2}$. As a matter of fact, Hicks concluded that the fringes of July 8th were of different type from those of the remaining days. Thus for his averages (in our Fig. 3) "the values of the ordinates are one-third of July 9 + July 11 - July 8 and one-third of July 9 + July 12 - July 8" (Hicks 1902) for noon and evening sessions respectively.

Let us finally compare with the interpretation that Michelson and Morley gave of their data. They start from the observation that "the displacement to be expected was 0.4 fringe" while "[…] the actual displacement was certainly less than the twentieth part of this". In this way, since the displacement is proportional to the square of the velocity, "the relative velocity of the earth and the ether is […] certainly less than one-fourth of the orbital earth's velocity". The straightforward translation of this upper bound is $\mathbf{v}_{\mathrm{obs}} < 7.5$ km/s. However, this estimate is likely affected by a theoretical uncertainty. In fact, in their Fig. 6, Michelson and Morley reported their measured fringe shifts together with the plot of a theoretical second-harmonic component. In doing so, they plotted a wave of amplitude $\mathbf{A_2} = 0.05$, that they interpret as *one-eighth* of the theoretical displacement expected on the base of classical physics, thus implicitly assuming $\mathbf{A_2}^{\mathbf{class}} = 0.4$. As discussed above, the amplitude of the classically expected second-harmonic component is *not* 0.4 but is just one-half of that, i.e. 0.2. Therefore, their experimental upper bound $\mathbf{A_2} < (0.4)/20 = 0.02$ is actually equivalent to $\mathbf{v_{obs}} < 9.5$ km/s. If we now consider that their estimates were obtained after superimposing the fringe shifts obtained from various sessions (where the overall effect is reduced) we deduce a substantial agreement with our result.

5. Summary of the classical ether-drift experiments and conclusions

The condensation of elementary quanta and their macroscopic occupation of the same quantum state is the essential ingredient of the vacuum of present-day elementary particle physics. In this description, one introduces implicitly a reference frame $\boldsymbol{\Sigma}$ where the condensing quanta have momentum $\mathbf{k} = 0$, which characterizes the physically realized form of relativity and could play the role of preferred reference frame in a modern re-formulation of the Lorentzian approach. To this end, we have mentioned in the Introduction some general theoretical arguments related to the problematic notions of a non-zero vacuum energy which induce to question the exact Lorentz-invariance of the vacuum state. These arguments suggest the possibility of a tiny vacuum energy-momentum flux, associated with an Earth's absolute velocity $\mathbf{v}$, which could affect the various forms of matter differently. Namely, it could produce small convective currents in a loosely bound system such as a gas or dissipate mainly by heat conduction with no appreciable particle flow in strongly bound systems as liquid or solid transparent media. In the former case, by introducing the refractive index $\mathbf{N}$ of the gas, convective currents of the gas molecules would produce a small anisotropy, proportional to $\mathbf{(N - 1)(v/c)^2}$, of the two-way velocity of light in agreement with the general structure Eq. (4) or with its particular limit Eq. (6). Notice that this tiny anisotropy refers to the system

S' where the container of the gas is at rest. In this sense, contrary to standard Special Relativity, **S'** might not define a true frame of rest. This conceptual possibility can be objectively tested by comparing with ether-drift experiments in *gaseous systems*. These experimental conditions are those adopted in the classical experiments by Michelson-Morley, Illingworth, Miller, Joos, … Modern experiments, in fact, have only been performed in vacuum where, anyhow, one expects the difference between Special Relativity and a Lorentzian interpretation to be at the limit of visibility. Now, with this premise, a complete re-analysis of all classical experiments was performed in Consoli *et al.* (2013). The final results are summarized in Tab. 2.

Experiment	gas in the interferometer	$v_{\rm obs}$(km/s)	v(km/s)
Michelson-Morley(1887)	air	$8.4^{+1.5}_{-1.7}$	349^{+62}_{-70}
Morley-Miller(1902-1905)	air	8.5 ± 1.5	353 ± 62
Kennedy(1926)	helium	< 5	< 600
Illingworth(1927)	helium	3.1 ± 1.0	370 ± 120
Miller(1925-1926)	air	$8.4^{+1.9}_{-2.5}$	349^{+79}_{-104}
Michelson-Pease-Pearson(1929)	air	$4.5 \pm ...$	$185 \pm ...$
Joos(1930)	helium	$1.8^{+0.5}_{-0.6}$	330^{+40}_{-70}

Tab. 2. The average velocity observed (or the limits placed) by the classical ether-drift experiments in the alternative interpretation of Eqs. (6), (8), (9)

As a summary of that work, we want to emphasize the following points:

1. An analysis of the individual sessions of the original Michelson-Morley experiment, in agreement with Hicks (1902) and Miller (1933), (see our Figs. 3 and 4), gives no justification to its standard null interpretation. As discussed in Sect. 3, this type of analysis is more reliable. In fact, averaging directly the fringe displacements of different sessions requires two additional assumptions, on the nature of the ether-drift as a smooth periodic effect and on the absence of systematic errors introduced by the re-adjustment of the mirrors on consecutive days, that in the end may turn out to be wrong.
2. One gets consistent indications from the Michelson-Morley, Morley-Miller, Miller and Illingworth-Kennedy experiments. In view of this consistency, an interpretation of Miller's observations in terms of a temperature gradient (Shankland *et al.* 1955) is only acceptable provided this gradient represents a non-local effect as in our picture where the ether-drift is the consequence of a fundamental vacuum energy-momentum flow. We have also produced numerical simulations of the Illingworth experiment in a simple statistically isotropic and homogeneous turbulent-ether model. This represents a zeroth-order approximation and is useful to illustrate basic phenomenological features associated with the picture of the vacuum as an underlying stochastic medium. In this scheme, Illingworth's data are consistent with

fluctuations of the velocity field whose absolute scale is well described by the value **v** = 370 km/s which today is used to describe the Earth's motion with respect to the Cosmic Microwave Background (CMB).

3. On the other hand, there is some discrepancy with the experiment performed by Michelson, Pease and Pearson (MPP) (Michelson *et al.* 1929). However, as discussed in Consoli *et al.* (2013), the uncertainty cannot be easily estimated since only a single basic MPP observation (Pease 1930) is explicitly reported in the literature. Therefore, since Miller's extensive observations (Miller 1933, Fig. 22), within their errors, gave fluctuations of the observable velocity in the wide range 4-14 km/s, a single observation giving $\mathbf{v}_{\text{obs}} \approx 4$ km/s cannot be interpreted as a refutation. This becomes even truer by noticing that the single session selected by Pease (1930), within a period of several months, was chosen to represent an example of extremely small ether-drift effect.
4. Some more details are needed to account for the Joos observations (Joos 1930).

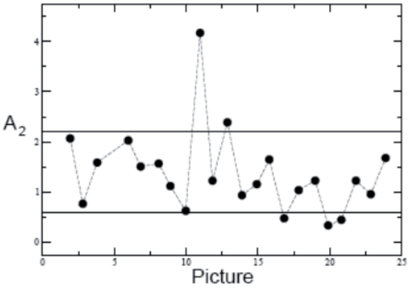
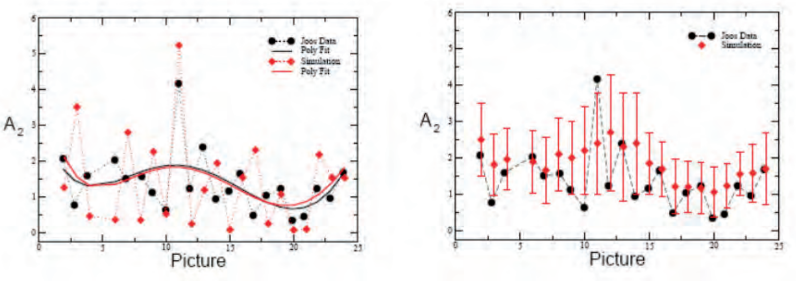


Fig. 6. The 2nd-harmonic amplitudes for all Joos' 22 observations, in units 10^{-3}. The vertical band between the two lines corresponds to the range $(1.4 \pm 0.8) \cdot 10^{-3}$

This experiment is particularly important since the data were collected at steps of 1 hour to cover the full sidereal day and were recorded by photocamera. For this reason, Joos' experiment is not comparable with other experiments (e.g. Michelson-Morley, Illingworth) where only observations at few selected hours were performed and for which, in view of the strong fluctuations of the azimuth, one can just quote the average magnitude of the observed velocity. Moreover, differently from Miller's, the amplitudes of all Joos' observations can be reconstructed from the published articles (Joos 1930) see Fig. 6. For these reasons, this experiment has deserved a more refined analysis and is central for our work. As discussed in Consoli *et al.* (2013), due to uncertainties in the original data analysis, the standard 1.5 km/s velocity value quoted for this experiment should be understood as an order of magnitude estimate and not as a true upper limit. Instead, our reported observable velocity $\mathbf{v}_{\text{obs}} \approx (1.8 \pm 0.5)$ km/s has been obtained from a direct analysis of Joos' fringe shifts. From this value, to deduce a kinematical velocity, one still needs the refractive index.

The traditional view, motivated by Miller's review article (Miller 1933) and Joos' own statements (Joos 1930), is that the experiment was performed in an evacuated housing. In these conditions, it would be easy to reconcile a large kinematical velocity $\mathbf{v} \approx 350$ km/s with the very small values of the observable velocity.

On the other hand, Swenson (1970) explicitly reports that fringe shifts were finally recorded with optical paths placed in a helium bath. Since Joos' papers do not provide any definite clue on this aspect, we have decided to follow Swenson's indications. In this case, by simply correcting in Eq. (9) with the helium refractive index $\mathbf{N} = 1.000035$ the result $\mathbf{v}_{\mathrm{obs}} \approx (1.8 \pm 0.5)$ km/s, one would get a kinematical velocity $\mathbf{v} \approx (217 \pm 72)$ km/s. However, as discussed in detail in Consoli *et al.* (2013), this is only a first partial view of Joos' experiment. In fact, by fitting Joos' experimental amplitudes to various forms of cosmic motion, one gets angular parameters which are very close to those that describe the CMB anisotropy, right ascension $\boldsymbol{\alpha} \approx 168$ degrees and declination $\boldsymbol{\gamma} \approx -6$ degrees. Still, to get a complete agreement, one should explain the absolute normalization of the amplitudes and the strong fluctuations of the data. Thus we have improved our analysis by performing various numerical simulations where the velocity components in the plane of the interferometer are not smooth functions but are represented as turbulent fluctuations whose Fourier components vary within time-dependent ranges which are controlled by the macroscopic parameters which describe the Earth's motion with respect to the CMB, say $(\mathbf{V}, \boldsymbol{\alpha}, \boldsymbol{\gamma})_{\mathrm{CMB}}$. Taking into account these stochastic fluctuations of the velocity field tends to increase the fitted average Earth's velocity, and can reproduce correctly Joos' 2nd-harmonic amplitudes and the characteristic scatter of the data, see Figs. 7 and 8 which are taken from Consoli *et al.* (2013).



Figs. 7-8. Joos' experimental amplitudes in Fig.6 are compared with numerical simulations of measurements performed at Joos' times. The stochastic velocity components are controlled by the kinematical parameters $(V,\alpha,\gamma)_{\mathrm{CMB}}$. In Fig. 7 (left), a single simulation is shown and the typical variation of each simulated entry is $(1 \div 4))\cdot 10^{-3}$ depending on the sidereal time. We also show two 5th-order polynomial fits to the two different sets of values. In Fig. 8 (right) we show the result of simulating the averaging process over 10 hypothetical measurements performed, at each Joos' time, on 10 consecutive days. The effect of varying the parameters of the simulation has been approximated into a central value and a symmetric error

In view of this consistency, we conclude that the range of kinematical velocities shown in Tab. 2 for Joos' experiment (which corresponds to the projection of the CMB motion at Joos' laboratory) is actually the most appropriate one. For this reason, the results reported in Tab. 2, besides providing an impressive evidence for a light anisotropy proportional to $\mathbf{(N\text{-}1)\ (v/c)^2}$, with the realistic velocity values $\mathbf{v} \approx 300$ km/s of most cosmic motions, could also represent the first experimental indication for the Earth's motion with respect to the CMB.

This gives a strong motivation to repeat these crucial measurements with today's much greater accuracy. To this end, let us now briefly consider the modern ether-drift experiments. In these experiments, the test of the isotropy of the velocity of light consists in measuring the relative frequency shift $\Delta\boldsymbol{\nu}$ of two orthogonal optical resonators (for a comprehensive review see Müller *et al.* (2003). Here, the analog of Eq. (8), is

$$\frac{\Delta\nu}{\nu_0} \approx (N-1)\frac{v^2}{c^2}\cos 2(\theta-\theta_0) \qquad (13)$$

where **N** is the refractive index of the gaseous medium filling the optical resonators and $\boldsymbol{\nu}_0$ their reference frequency. Testing this prediction requires replacing the high vacuum usually adopted within the optical resonators with a gaseous medium and studying the substantially larger frequency shift introduced with respect to the vacuum experiments.

This substantial enhancement is confirmed by the only modern experiment that has been performed in similar conditions: the 1963 experiment by Jaseja at MIT (Jaseja *et. al.* 1964). For a proper comparison, one has to subtract preliminarily a large systematic effect of about 270 kHz that was present in the data and interpreted by the authors as probably due to magnetostriction. As suggested by the same authors, this spurious effect, which was only affecting the normalization of the experimental $\Delta\boldsymbol{\nu}$, can be subtracted by looking at the variations of the data. In this case, the residual variations of a few kHz are consistent (Consoli, Costanzo 2004a,b), see Fig. 9, with the refractive index $\mathbf{N} \approx 1.00004$ of the He-Ne mixture and the typical variations of the kinematic Earth's velocities reported in Tab. 2.

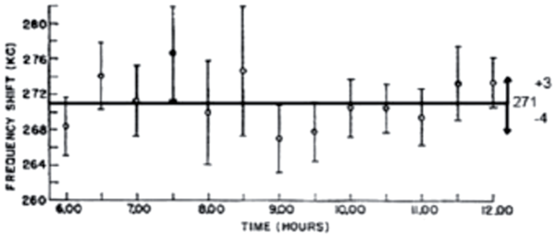


Fig. 9. Plot of relative frequency variation of two masers with 90° rotation as a function of the time of day between 6:00 a.m. and 12:00 noon on 20 January, 1963

Therefore we conclude that the non-trivial level of consistency of our analysis, by itself, motivates the new generation of precise laser interferometry experiments in gaseous media. Apart from the potential important implications, without such definite clarification, the classical ether-drift experiments would remain as an inexplicable enigma for both physics and history of science.

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