A program of research for discovering an alternative formulation of quantum mechanics

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Abstract: I recall my proposal of a general strategy for discovering an alternative formulation to Dirac-von Neumann’s formulation of Quantum Mechanics and moreover I review its advancements. First, I have planned to study what are the foundations of classical physics. I found out that they are constituted by two dichotomies concerning respectively Mathematics and Logic. I made use of them for characterizing the basic features of the alternative formulation of Quantum Mechanics to be discovered, first of all the alternative choices on the two dichotomies to those of both Newton’s Mechanics and the dominant formulation of Quantum Mechanics. Subsequently, I have followed two sub-strategies. The former one was to discover, as its first step, the alternative formulations of the main classical theories. I have obtained them. The second step, to be still performed, is to rationally re-construct on this alternative basis and according to the two dichotomies the historical birth of Quantum Mechanics. The latter sub-strategy is to scrutinize all the already suggested formulations of Quantum Mechanics in order to inspect whether one of them enjoys the features of the wanted alternative formulation. No one formulation satisfies such features, although several formulations closely approach the alternative one; a rigorous re-formulation of them according to the alternative choices on the two dichotomies requires a sophisticated technical competence. By the way, a new history of both the entire classical physics and part of modern physics has been obtained according a new point of view of a pluralist nature.

Keywords: Quantum Mechanics, dichotomies in Mathematics and Logic, alternative formulations of classical physical theories, alternative quantum mechanics.

1. The recognition of the foundations of theoretical physics

In 1976, at Lecce Conference on *Science and society* I stated a program of research for discovering an alternative formulation of QM (this proposal was not recorded by the proceedings (Donini *et al.* 1977) of that conference). I suggested that before tackling the foundational problems of modern physics, it is necessary to further explore classical physics – even the inertia principle – in order to accumulate more knowledge on the foundations of physics in general, and then to apply them to the study of QM.

After two decades my investigations on classical physics have obtained a first result, i.e. a clear-cut definition of the foundations of physics.

An analysis of past historical theories in Logic, Mathematics, Physics and Chemistry has suggested that not only – as it is well-known – the foundations of science are constituted by Logic and Mathematics, but in addition each of them has to be conceived in a dichotomic way, either constructive mathematics or classical mathematics and either classical logic or non-classical logic, respectively; or even, in corresponding philosophical terms, either potential infinity (PI) or actual infinity (AI), and either the deductive organization of the theory as derived from few axioms (AO), or the organization of a theory which is aimed at solving a basic problem (PO), respectively. In particular, I obtained the ideal model of a PO theory by means of a comparative study of the main non-deductive scientific theories (Drago 2012).

By means of the two dichotomies I have characterized Newton's mechanics as based on the choice AI owing to its use of infinitesimal analysis, and the choice AO, owing to its derivation of all mechanics laws from three principles (Drago 1988).

I have also characterized how the formulations of this theory – for instance, the three formulations suggested by L. Carnot, Lagrange and Hamilton – differ from Newton's formulation. They differ not only in making easier the resolutions of the problems – as even Mach has maintained (Mach 1893, part IV, chapter III), but first of all in their basic choices (and, as a consequence, in their principles, mathematical techniques and notions).

The two dichotomies suggest also a new characterization of the Dirac-von Neumann (DvN) formulation of QM. It is easily recognised that DvN is based (apart its measurement process) on the choices AI, owing to its use of the highly sophisticated mathematics of Hilbert space, and AO, owing to the a priori role played by this same notion inside the theory (Drago 1991a). Surprisingly, these choices are the same of Newton's mechanics, exactly that theory whose foundations have been contested by the scientists who started modern physics. This paradox constitutes the main criticism to the DvN, since its choices represent a backward theoretical attitude.

As a second step of my general strategy, I have approached the problem of looking for an alternative formulation of QM according to the two following sub-strategies:

a. a study aimed at discovering the alternative formulations of all classical theories and then rationally re-constructing on their basis and according to the two dichotomies the historical birth of QM; from this re-construction the alternative formulation to DvN is expected to result;

b. a recognition of all previously suggested formulations of QM in order to recognize the wanted alternative formulation, or at least to recognize those formulations which are less far from the wanted result.

3. Rationally re-constructing QM from the alternative formulations of its basic classical theories: the recognition of all these alternative theories

Mechanics. I have discovered the alternative formulation to Newton's. As a fact, L. Carnot declares in philosophical terms (Carnot L. 1783, pp. 101-103; 1803, pp. xiii-xvii, 2-3) that his formulation is founded on exactly what at present we know as the choices PI and PO. He declares (Carnot L. 1803, p. x) also that his theory is based on the principle of virtual works (PVW). Actually, L. Carnot's theory generalizes previous Leibniz' theory of the impact of only elastic bodies by deriving it from the PVW and in addition introducing an index of elasticity for considering the impact of all kinds of bodies. His great theoretical novelty is its basic problem, i.e. the search for the invariants of an impact of bodies; for the first time in the history of theoretical physics he derives them; its mathematical technique is surprisingly an elementary algebraic one (Carnot L. 1783, pp. 44ff; Drago 2004). I have accurately re-formulated this theory upon the principle of the *impossibility* of a motion *without* an end (IPM), which in a PO theory is the usual methodological principle from which a non-classical reasoning starts. From this principle the PVW easily follows and from here the entire development of L. Carnot's theory (Bellini, Drago, Mauriello 2007).

Thermodynamics. By extending this alternative formulation of mechanics to heat engines, Lazare's son, Sadi, founded thermodynamics. Owing to his purpose of addressing his book to people not educated to higher mathematics, his theory does not make use of calculus – apart in a footnote. Hence, the choice on the kind of mathematics is PI. The other choice is PO since the theory is based on a problem of the highest efficiency in the conversions of heat in work (Carnot S. 1824, p. 4). In addition, in a surprisingly consistent way he makes use of non-classical logic through doubly negated propositions (DNPs) which are not equivalent to the corresponding affirmative ones for lack of evidence of the latter ones; through them the author composes indirect proofs (Drago 1991b; Drago, Pisano 2000). Yet, textbooks' formulation, relying on the concept of the conservation of energy, differs from Sadi Carnot's one, which relies on a discarded concept (caloric). However, Drago and Pisano (2013) made apparent its full linkage with father's theory by deriving Sadi Carnot's valid results from the PVW. In addition, Callen (1974) has proved, although by applying to statistical mechanics Lagrange's formulation[1] rather than L. Carnot's one, that "thermodynamics is the science of symmetries". This fact confirms what is obtained by a comparative analysis of most physical theories, i.e. the characteristic mathematical technique of a PO theory is the symmetry (Drago 1996).

Electromagnetism. I have interpreted through the two dichotomies the history of its birth (Drago 2003). The early theory was built on both Carnotian-like notions (most of which have been suggested by Faraday) and Newtonian-like notions (which have been advocated by other scientists). Subsequently Maxwell has suggested a theory based on his celebrated equations, from which all electromagnetic laws are derived; hence its choice is AO. In contrast to Faraday's rejection of usual mathematics, these equations

[1] Lagrange's theory is based on the choice PO (the problem is how obtain the solution of whatsoever problem in mechanics) and the choice AI (it makes use of infinitesimal analysis) (Drago 2006).

have been obtained by means of that sophisticated mathematics which is necessary for representing the fluid dynamics of vortices. No surprise if Maxwell's equations, concerning differential equations on local quantities, have been considered an expression of the usual infinitesimal calculus, which is AI. Indeed, Maxwell's contemporaries thought that his theory had confirmed, although through different terms, the Newtonian paradigm (whose choices are the same, AI&AO).

Rather, an alternative theory of electromagnetism may be based upon Faraday's problem (PO) of how joining together an electricity theory with a magnetism theory through integral laws on global quantities (PI). This is the phenomenological electromagnetism (Rossi 1957, p. 303). Moreover, at present we know that constructive mathematics can express also Maxwell's equations because it includes differential operations; hence, according to the choice PI one can include in the alternative formulation also Maxwell's equations. An essential result of Maxwell's equations is the electromagnetic wave equation, from which both the formula for the light velocity and the theory of optics are derived. Yet, this equation cannot be solved in general by constructive means (Pour-El, Richards 1989).[2] However the same equation can be solved when the energy is bounded, that is a natural limitation for an operative, phenomenological theory. Hence also the alternative formulation of electromagnetism includes this wave equation; it may be derived from the given global formulas by testing in a heuristic way a wave function as their solution (Sànchez del Rio 1991).

Let us recall that a PO theory is based on the mathematical technique of symmetries; in this case the Lorentz' group is in question.

It was Einstein's paper (1905b) on special relativity that has proved the invariance of Maxwell equations; the method is appropriate to a PO theory: it is heuristic and it is based on a limitation principle, $v \leq c$, suggested to him by the IPM in thermodynamics (Zahar 1989, pp. 113-121). Moreover, its mathematics is at all constructive.

Statistical mechanics. Its simpler part, the kinetic theory of gases, has been surprisingly recognized to be a generalization of Leibniz-L. Carnot's mechanics, which I recall is based on the alternative choices PI&PO. This fact is confirmed by its history; the new theory started only when it was based – first by Leibniz' follower, D. Bernoulli, and a century after by Wallastone, Clausius and Maxwell – on the alternative notions to the Newtonian ones (Drago 2014).

Both Boltzmann and Gibbs have founded statistical mechanics upon Hamilton's mechanics, which in general is not solvable in constructive mathematics.[3] Einstein (1902; 1903; 1904) gave a new foundation of this theory according to an approach of kinetic theory of gases, which we know derives from L. Carnot's mechanics. Einstein based the theory upon the conservation of energy, which is a result of L. Carnot's

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[2] These authors and some other scholars have looked for an essentially non-constructive QM theorem. Billinge (1997) has shown that even Gleason's theorem has a constructive counterpart – that does not means that it is constructive. However, no scholar has made reference to the different formulations of a same theory. Already in 1982 I have proved that the question depends from the particular formulation one chooses as a representative of a physical theory (Drago 1982). I have also suggested a general method for inquiring the constructiveness of a physical theory (Drago 1986).

[3] Da Costa and Doria (1991) have proved that Hamilton's formulation is undecidable in constructive mathematics. I studied the obscure foundations of Statistical mechanics in (Drago 2016a).

mechanics.[4] This principle of Einstein's theory is of course decidable and moreover the following theoretical development is constructive because one can intend with impunity the differentials as finite difference quantities. (Yet, a further analysis deserves Einstein's use of Liouville's theorem).[5]

Table 1 summarizes the situation of the accumulated results; it shows that there exists an alternative formulation to each classical theory that is involved in the birth of QM.

	PI	**Constructive Mathematics**	**PO**	**Non-classical Logic**	**First Principles Limitation**	**Invariants**
Mechanics	LC	LC	LC*	LC*	IPM-PVW	LC (spatial)
Thermodynamics	(SC)/CKC	(SC)/CKC	(SC)/CKC*	(SC)/CKC*	IPM-PVW	Callen
Electromagnetism	Ph.	Ph.	Faraday	Faraday?	?	Lorentz
Kinetic theory of gases	LC	LC	LC	Prob.?	¬(L> 0)	Callen
Statistical Mechanics	Einstein	Einstein	Einstein	Prob.?	¬(L> 0)	Callen

Table 1. The alternative formulations of the main classical theories
Legenda: LC = Lazare Carnot's formulation; SC = Sadi Carnot's formulation; CKC = Carnot-Kelvin-Clausius' formulation; IPM = Impossibility of a perpetual motion; PVW = Principle of virtual works; * = in a more clear version by means of improvements of the original theory. Prob.? = hintes that the introduction of probabilities surrogates the use of non-classical logic

4. Rationally re-constructing QM from the alternative formulations of its basic classical theories: the attempts to rationally re-construct the birth of QM

In order to proceed with the latter step of the former sub-strategy, one can anticipate the result by characterizing six features of the alternative formulation of QM which are derived by both the two dichotomies and the alternative features of the above classical theories: *i*) its organization is PO; *ii*) its prime principle is a limitation principle, of course the uncertainty principle; *iii*) its way of reasoning pertains to non-classical logic; *iv*) its mathematics is the constructive one; *v*) its mathematical technique is that of symmetries; *vi*) its limit for $h \rightarrow 0$ has to give Lazare Carnot's formulation of mechanics, or, more in general, a theory relying on an impossibility principle, as the PVW is. In addition, the following *à la* Koyré's propositions which characterize the alternative theories to Newton's one suggest the following change in the basic notions from the Newtonian paradigm to the alternative formulation of QM:[6] "Evanescence of the force-cause and discreteness of matter [and even light]."

[4] The usual starting point for the non-classical reasoning in the alternative theories - i.e. the principle of the *impossibility* of a motion *without* an end - may be substituted by the following doubly negated principle: "*No positive* (= *not* null) work from gas motion and from constraints' reactions".
[5] Notice the discovery of Einstein's treatment by Peliti and Rechtman (2017).
[6] They have been obtained by interpreting through the two Newtonian choices the original Koyré's ones and then constructing on the alternative choices those propositions which are valid for the classical alternative theories (Drago 1994).

My studies under the light of the two dichotomies have obtained that in the 1905 paper (Einstein 1905c), *i*) Einstein has built a PO theory (in his words, a "principle theory") because it s aimed at solving the problem of existence of quanta; *ii*) he has appropriately made use of non-classical logic (although by mere ingenuity); *iii*) in the foundation of the entire theoretical physics he has overtly recognized the dichotomy on the infinity ("continuum" vs. "discrete") and *iv*) he has chosen "the discrete", i.e. the PI (Drago 2013). In other words, in his paper Einstein has almost exactly recognized the alternative choices, PO and PI, to Newton's. No surprise if in the early years of the XX century Einstein's 1905 paper only has represented a qualified answer to the theory of the blackbody; surely, this clever theoretical attitude derives from his attributing an alternative character (limitation principle, the elementary mathematics) to thermodynamics, that we know has the alternative choices PO and PI. As a fact, Einstein has declared this paper his "more revolutionary paper" (Einstein 1905a).

Yet, Einstein's method, i.e. an analogy between a gas of particles and a "gas" of quanta, cannot be exactly reiterated in other case studies. In the following years various parts of QM have been obtained from analogies between the new theoretical situations and suitably chosen parts of classical theories (Darrigol 1992). Bohr has suggested that these analogies result from the applications of a "principle of correspondence". This principle was never "proved"; it remained a merely intuitive principle. By means of the two dichotomies one may give reason for this negative outcome. The classical theories taken in consideration all rely on AI&AO, whereas the early QM relies on the choices PO and often PI; the different choices allow to link the two theoretical parts by no more than local and occasional analogies. As a verification, the historical sequence of all the analogies exploited during the process of QM's construction does not alludes in any way to the four choices on the two fundamental dichotomies.

Let us now consider a formal way to re-construct the birth of QM. It is an encouraging fact that L. Carnot's mechanics may be in a natural way extended to be invariant under Lorentz' group (Drago 2001; Scarpa 2002). However, it is well known that the resulting theory, i.e. special relativity, is incompatible with QM, owing to the non linearity of the quantum commutation relationships. Hence, this theoretical path from L. Carnot's formulation to QM, is barred.

At present, it is still an open problem how re-visiting according to the two dichotomies the historical paths leading to QM.

5. The strategy for recognizing an alternative formulation of QM among those already invented

Instead of following the above laborious sub-strategy, one may hope to find out the alternative formulation to the DvN by discovering, or at least by suitably re-interpreting, according to the alternative couple of choices PO&PI, a formulation of QM which has been already suggested by quantum theorists. I have explored under the light of the two dichotomies four classes of (broadly intended) formulations of QM.

Alternative formulation before the birth of DvN. Fortunately such formulation there exists. Heisenberg's matrix mechanics has been surely founded on the alternative choices, because it is aimed at solving the problem of the atomic spectra (PO) and it makes use of an algebraic, hence constructive, mathematical technique (PI). Yet, this formulation is incomplete and it cannot appropriately make use of continuous operators (Beller 1983, p. 475). The completion of the original formulation independently from the theoretical framework of DvN (which includes Heisenberg theoretical suggestions as a mere "Heisenberg picture") constitutes an open problem.

Formulations closely approaching the wanted alternative. I have looked for all existing formulations of QM (around 26) that have been by ingenuity invented by theoretical physicists. It is not easy to decide which are the basic choices of each one, because a theory which at glance appears as based on AI, a more accurate analysis may interpret it as belonging to constructive mathematics; and moreover a theory which was suggested in an AO-style may hid a PO theory. Therefore, in a previous paper I have contented myself with rough characterizations of the choices of each of the above formulations of QM. Among them I have found two formulations which seem to be founded upon the alternative choices PO and PI, i.e. T.F. Jordan's (1985) and Bub's (2005). However, the former formulation is incomplete and the latter one is unsatisfactory for it relies on a non-physical notion, information (Drago 2016c).

Re-formulating QM upon an accurate definition of quantum logic. By means of the two dichotomies I have re-constructed the history of QM as a progressive recognition of the alternative notions, techniques and principles. I obtained an astonishing theoretical progression marked by a sequence of five-years steps (Drago 2002). In addition, this progression makes apparent that the discovery – through Birkhoff and von Neumann's seminal paper (1936) – of a non-classical logic in the foundations of QM has occurred after the construction of both its first two formulations and also the DvN. This fact proves that *i*) by lacking of an essential aspect of a PO theory, i.e. the non-classical logic, Heisenberg's matrix mechanics was doomed to be absorbed into the DvN framework (which is founded on the dominant classical logic); *ii*) owing to its classical logic, the DvN cannot be considered as an adequate formulation of QM; *iii*) the discovery of an alternative formulation of QM through the new logical basis is needed. Several theorists wanted to accurately define the exact quantum logic in order to re-construct upon it QM. Actually, Birkhoff and von Neumann have deliberately excluded the intuitionist logic; which is instead the appropriate logic of a PO formulation of QM, whose main problem is how measure the state of the system notwithstanding the indeterminacy relationships. Hence, all attempts to define a new kind of non-classical logic for QM, being performed outside of both a PO and the well-founded (since the 1930's) intuitionist logic, are doomed to fail, as eighty years of unsuccessful attempts prove it. A first attempt of re-constructing QM inside a well-defined PO formulation and by making use of DNPs is the paper (Drago, Venezia 2002).

Any already suggested formulation of QM that relies on the symmetry technique? Hermann Weyl's book – aimed at formulating QM through symmetries (Weyl 1930) – appears as the fortunate answer to the above question. Yet, a closer analysis of this formulation shows that its theoretical development relies upon

Schrödinger's mechanics, whose basic choices are easily recognized as AO (owing to his a priori notion of the amplitude of probability) and AI (as each second order differential equation); moreover, in the initial development of Weyl's theory the mathematics is an algebra of finite groups; yet, subsequently this mathematics is extended in an informal way to continuous groups (Drago 2000). In order to rigorously re-formulate this attempt according to the choices PI and PO a competence in the most sophisticated mathematics of the classical and constructive group theory is required. A first attempt for obtaining a QM based on symmetries has been presented some years ago (Drago, Pirolo 1997).

Table 2 summarizes the previous results and some open problems.

	PI	Constructive Mathematics	PO	Non-classical Logic	Symmetries
Einstein's paper	+*	+*	+*	+*	-
Correspondence Principle	-	-	+	+	-
Matrix Mechanics	+	+	+	-	-
Weyl's formulation	(+)	(+)	(+)	-	+
Quantum Logic	-	-	(+)	(+)	-

Table 2. Formulations closely approaching the alternative one
Legenda: (+) = not precisely expressed; * = almost accurately expressed; - = lacking feature

6. Conclusions

The program of research suggested by my proposal of forty years ago is successful in having obtained: *i*) the first wanted result, i.e. discover the foundations of theoretical physics *ii*) the first step of the former sub-strategy, i.e. recognition of the alternative formulations to each theory of classical physics involved in QM's birth; *iii*) the recognition of the basic features of the wanted alternative formulation of QM (however, the reformulation, planned by the first sub-strategy, of the historical path leading from classical physics to the alternative formulation of QM is at present an unsolved problem); *iv*) no scholar has by ingenuity suggested a complete formulation of QM based on PI&PO; *v*) many partial results concerning the second sub-strategy, i.e. the foundational relevance of Einstein's first theory on the existence of quanta, Heisenberg's matrix mechanics (the first formulation of QM), Weyl's group theory formulation of QM, plus two other formulations (Jordan's and Bub's); all together these formulations represent a close approximation to the wanted formulation of QM relying on the two alternative choices; which nevertheless has not still been discovered.

However, all the above facts show that the four basic choices of the two dichotomies catch the novelty of QM and moreover that the two dichotomies have been perceived by some prominent scientists. Hence, the above-illustrated foundations of physics are constitutive of the evolution of modern physics and can appropriately tackle the problem of suggesting an alternative formulation to the DvN.

By the way, all these results have suggested a more detailed interpretation of the history of both classical physics and the births of the two main theories of modern physics. This interpretation corresponds to a pluralist point of view, which is a novelty in the historiography of Physics.

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