Particle tracks in a cloud chamber: the Mott's conjecture (1929)

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Abstract: Since the very early days of quantum mechanics, questions and theoretical proposals concerning the observed tracks left by atoms or sub-atomic particles in a cloud chamber produced a lively debate. In fact, tracks are interpreted as macroscopic footprints of the passage of a quantum particle but are perfectly described as trajectories of a classical particle in a classical magnetic field. In an almost unnoticed paper written at the end of the twenties, Mott investigated, in perturbation theory, the Schrödinger evolution of a particle emitted by a radioactive source inside a simplified quantum environment. He was able to prove that successive ionizations of atoms of the environment are more likely if atoms lie on the same line containing the radioactive source. His line of reasoning did not make use of any reduction of the particle wave packet due to the interaction with a measurement apparatus. I will summarize recent attempts to analyze the problem in terms of the so-called environment induced decoherence.

Keywords: Cloud chamber, wave packet reduction, decoherence.

1. Introduction

The distinguished English physicist Sir Nevill Francis Mott (Leeds, 30 September 1905-Milton Keynes, 8 August 1996) won the Nobel Prize in Physics in 1977 for his work on the metal-non-metal transition and, more generally, on the electronic structure of disordered systems. He shared the award with P.W. Anderson and J.H. van Vleck. For further details of his biography one can consult Pippard's biographical memoir (Pippard 1998), which is a mine of information about Mott's intense scientific life.

His third scientific article (Mott 1929) was published in the Proceedings of the Royal Society in 1929 at a time when he was a young Lecturer at the University of Manchester. Mott wrote it at the dawn of Quantum Theory aiming at clarifying the meaning of wave-particle duality and the role of the measurement process in the atomic and sub-atomic systems dynamics. The paper remained little known inside the Physics community, as Pippard indirectly confirms mentioning it, mainly because "the paper was considered sufficiently original to be included in a rather recent collection devoted to the quantum theory of measurement (Pippard 1998, p. 322).[1] In fact, Mott's

[1] Pippard refers to (Wheeler, Zurek 1983).

contribution should be considered the earliest innovative attempt to lay the foundations of the theory of environment-induced decoherence, i.e., the dynamical mechanism responsible for the transition to a classical behavior of a quantum particle as a consequence of its interaction with the environment.

In this short note, I will try to sketch few noteworthy aspects of the controversy about the theoretical explanation of the tracks observed in a cloud chamber. The debate went on between the Fifth Solvay Conference, held in October 1927, and the year 1932 when the "Copenhagen" formulation of Quantum Mechanics was given its final form by von Neumann. For more detailed studies on historical and technical perspectives of the subject see (Figari, Teta 2012, 2014; Dell'Antonio *et al.* 2015).

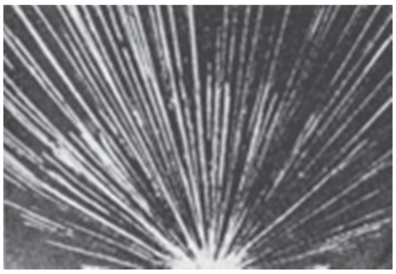

Fig. 1. Particle tracks in a Wilson's cloud chamber

2. The Cloud Chamber

The first tracking chamber, fit for the purpose to detect tracks of sub-atomic particles, was invented and put in operation in 1911 by C.T.R. Wilson.[2] In his own words,

> In the first years of the XX century […] ideas on the corpuscular nature of alpha and beta rays had become much more definite, and I had in view the possibility that the track of an ionizing particle might be made visible and photographed by condensing water on the ions which it liberated (Wilson 1927).

In order to track the ionizing radiation, Wilson utilized a cavity containing a mixture of air and water vapor brought into a super-saturated state by a rapid expansion lowering its temperature. The α-particles were released by a radioactive source inside the chamber and induced ionization of the vapor molecules that, in turn, operated as condensation nuclei for the formation of drops of water. The sequence of drops was instantly photographed, making visible the α-ray trajectories. There was no doubt about the interpretation of the tracks as trajectories of the α-particles. In fact, they were accurately described as trajectories of a charged classical particle in a classical electromagnetic field.

[2] For details on the Wilson's experimental apparatus see (Leone, Robotti 2004).

For the astonishing clarity of the experimental outputs, Ernest Rutherford described the Wilson's cloud chamber as the most original and wonderful instrument in scientific history.

Elaborated in the years 1925-1927, the standard formulation of Quantum Mechanics appeared to be in open contrast with the classical description of the tracks outlined above. Considered the difficulties of the old Quantum Theory to connect the frequencies emitted by atoms with the revolution period of any supposed electronic orbit, Heisenberg sought to formulate a theory avoiding the concept of electron orbit, a concept "which I had expressly forbidden myself" (Heisenberg 1983). He succeeded in this task in a way that disoriented also his co-workers: "After my return to Gottingen I showed the paper to Born, who found it interesting but somewhat disconcerting, inasmuch as the concept of electron pathways was totally eliminated" (Heisenberg 1983).

Heisenberg, Born and Jordan finally formulated Matrix Mechanics characterized by an explicit refusal to consider of any relevance classical kinematic concepts like position, velocity or trajectory in order to describe the atomic structure. The main reason for this conclusion is that position and velocity are quantities that can barely be observed at atomic level, whereas a proper physical theory should always rely on observable quantities.

Following a totally different line of thought, Schrödinger found out that it was possible to maintain a space-time description at the expense of describing microscopic objects as waves instead of point particles. The theory he formulated took the name Wave Mechanics. On the basis of the analogy between Optics and Mechanics, Schrödinger was able to derive the evolution equation for the wave $\psi(x,t)$. Moreover, he proposed a first physical interpretation of $e|\psi(x,t)|^2$ as the charge density at the point x and time t of the electron with total charge e, but the proposal was rapidly rejected due to the fact that, in general, the solutions of the evolution equation spread in space as time goes by.

The finally accepted interpretation was given by Born (1926). According to his proposal, $|\psi(x,t)|^2$ is interpreted as the probability density to find the object in x at time t. After Born, Quantum Mechanics has been accepted as a theory that can only provide probabilistic predictions (of the position or of any other observable relative to a microscopic object).

Bohr attempted to harmonize Wave and Matrix Mechanics in a unified and consistent description of atomic phenomena. The occasions were the lecture delivered at the congress in Como, on September 1927 (Bohr 1927), and the general discussion in the subsequent Fifth Solvay Conference in Brussels (Bacciagaluppi, Valentini 2009). Bohr's approach soon became the core of the so-called Copenhagen or standard interpretation of Quantum Mechanics.

It is worth mentioning here the crucial role played in the standard interpretation by the act of measurement.

- A measurement apparatus must be considered as a classical object.
- The result of a measurement is the determination of one of the possible complementary properties of the quantum system.

- The property (which is incorrect to be thought as pre-existing) is produced only as the result of the interaction with the classical apparatus. The instantaneous change of the system state in the measurement process is denoted as wave packet reduction (or collapse).

The axiomatic formulation of the measurement process described above is the most controversial aspect of the Copenhagen interpretation. It has raised a long debate that still continues.

Few points open to question are worth mentioning. First, it is not explained why the experimental device, despite being made of atoms, should behave as a classical object. Moreover, it is not clear where the borderline between the measurement apparatus (characterized by a classical behavior) and the system (characterized by a quantum behavior) should be put. The problem is usually solved pragmatically in each specific situation but, at a conceptual level, the ambiguity remains. Finally, one has to renounce to the universality of the dynamical law. In 1932 von Neumann formalized this last feature of the theory postulating two different kinds of evolution for the system: a genuine quantum evolution governed by the Schrödinger equation when the system is not measured and a sudden stochastic evolution corresponding to the wave packet reduction when the system is measured.

3. The debate on the tracks in a cloud chamber

According to the first theoretical analysis of the radioactive decay given by Gamow (1928), the emitted α-particle must be described by a wave function having the form of a spherical wave, with center in the radioactive nucleus and isotropically propagating in space. In fact, the initial isotropy is also suggested by the isotropy in the experimental output (see Fig. 1). Therefore, the non-trivial problem arises as to how such an initial spherically symmetric wave packet can produce the observed classical trajectories.

The theoretical explanation of the observed tracks in a cloud chamber was already approached by Born in 1927 during the general discussion at the Solvay Conference. In his words:

> Mr. Einstein has considered the following problem: A radioactive sample emits α-particles in all directions; these are made visible by the method of the Wilson cloud chamber. Now, if one associates a spherical wave with each emission process, how can one understand that the track of each α-particle appears as a (very nearly) straight line? In other words: how can the corpuscular character of the phenomenon be reconciled here with the representation by waves? (Bacciagaluppi, Valentini 2009, pp. 147-149).

According to Born, the explanation has to be connected to the "reduction of the probability packet". The effect of each vapor atom ionization in the chamber is described as follows:

> As soon as such ionization is shown by the appearance of cloud droplets, in order to describe what happens afterwards one must reduce the wave packet in the immediate vicinity of the drops. One thus obtains a wave packet in the form of a ray, which corresponds to the corpuscular character of the phenomenon" (Bacciagaluppi, Valentini 2009).

According to this point of view, the interaction of the quantum system (the α-particle) with a classical measurement apparatus (the atoms of the vapor) produces "reduction" of the spherical wave to a wave packet with definite position and momentum.

Heisenberg considered the cloud chamber problem in his lectures at the University of Chicago in 1929 (Heisenberg 1930). Through an exhaustive qualitative investigation of the problem made according to the standard interpretation of Quantum Mechanics, he reached the same conclusions as Born. For many years, his analysis of the phenomenon has been considered the most convincing and clear solution to the problem of the tracks in a cloud chamber by the majority of the Physics community.

In 1929 C.G. Darwin addressed a problem of collision between quantum particles, fully inside the framework of Wave Mechanics, with the aim to "take a problem which would be regarded at first sight as irreconcilable with a pure wave theory, but thoroughly typical of the behavior of particles, and show how in fact the correct result arises naturally from the consideration of waves alone" (Darwin 1929). He emphasizes that if a quantum particle in interaction with the large number of quantum particles making up a macroscopic environment (e.g. a measurement apparatus) is considered, one has to take into account that the wave function of the entire system is not a wave in ordinary three dimensional space but rather it is a function of the coordinates of all the particles.

According to this point of view, when addressing the problem of the evolution of an α-particle in the cloud chamber, one should consider that the wave function ψ is a function of the coordinates of the α-particle and of the coordinates of the atoms in the chamber. In particular, before the first collision, it is a product of the spherical wave for the α-particle times a set of stationary (in general ground) states for the atoms. "But the first collision changes this product into a function in which the two types of coordinates are inextricably mixed, and every subsequent collision makes it worse" (Darwin 1929).

Such complicated function contains a phase factor and "without in the least seeing the details, it looks quite natural to expect that this phase factor will have some special character, such as vanishing, when the various co-ordinates satisfy a condition of collinearity" (Darwin 1929). He concludes: "So without pretending to have mastered the details, we can understand how it is possible that the ψ function, so to speak, not to know in what direction the track is to be, but yet to insist that it should be a straight line" (Darwin 1929).

4. Mott's paper

In his seminal paper of 1929, Mott concretely realized the program enunciated by Darwin. In the introduction, Mott recognizes to have been inspired by Darwin's paper in his attempt to explain the typical particle-like properties of an α-particle in a cloud chamber using only Wave Mechanics. He admits that such a point of view seems at first sight counterintuitive, since "it is a little difficult to picture how it is that an outgoing spherical wave can produce a straight track; we think intuitively that it should ionize atoms at random throughout space" (Mott 1929). Like Heisenberg, Mott points out that the crucial point is to establish the frontier between the system under consideration and the measuring apparatus. He points out that there are two possible approaches: either one considers the α-particle as the quantum system under consideration (and the gas of the chamber as the measuring device) or one chooses to investigate the quantum system consisting of the α-particle and of the atoms of the gas. Mott proceeds toward a detailed analysis of the problem following closely the latter approach.

He claims that the intuitive difficulty mentioned above can be overcome since it arises from our erroneous "tendency to picture the wave as existing in ordinary three dimensional space, whereas we are really dealing with wave functions in multispace formed by the co-ordinates both of the α-particle and of every atom in the Wilson chamber" (Mott 1929).

The model considered by Mott consists of the α-particle, initially described by a spherical wave centered at the origin, and the electrons of two hydrogen atoms initially in their ground states. The main result of the paper can be summarized in the following statement: the two hydrogen atoms have negligible probability to be both excited unless the atoms and the radioactive source lie on the same straight line. The result, obtained only analyzing the Schrödinger dynamics of the entire system and without having any recourse to wave packet reduction, implies that only straight tracks have non zero probability to be observed in the cloud chamber camera.

From an historical point of view, it would be interesting to investigate the reasons why the line of research initiated by Mott was not further developed and remained almost neglected for many years. One reason could be the influence and the authority of the position expressed by Born and Heisenberg. The consequence has been to discourage the new approach to the problem, with the motivation that it was ineffectively more complicate without giving real advantages from the conceptual point of view.

The astonishing experimental progresses made in the last decades of previous century made possible a detailed examination of the classical/quantum border. Those progresses have fostered new theoretical investigations on the interaction between a quantum system and a quantum environment and on the quantum to classical transition. A general feature characterizing all these investigations is the attempt to avoid the use of any form of wave packet reduction.

References

Bacciagaluppi G., Valentini A. (2009). *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference.* Cambridge: Cambridge University Press.

Born M. (1926). “Zur Quantenmechanik der Stossvorgänge”. *Zeitschrift für Physik*, 37, pp. 863-867.

Bohr N. (1928). “The quantum postulate and the recent development of atomic theory”. *Nature*, 121, pp. 580-590.

Darwin C.G. (1929). “A collision problem in the wave mechanics”. *Proceedings of the Royal Society of London*, A 124, pp. 375-394.

Dell’Antonio G.F., Figari R., Teta A. (2015). *Classical-Like Trajectories of a Quantum Particle in a Cloud Chamber*, in Blanchard Ph. and Frölich J. (eds.), *The Message of Quantum Science – Attempts Toward a Synthesis.* Heidelberg: Springer (Lecture Notes in Physics, n. 899).

Figari R., Teta A. (2012). “Emergence of classical trajectories in quantum systems: the cloud chamber problem in the analysis of Mott (1929)”. *Archives for the History of Exact Sciences*, 67, pp. 215-234.

Figari R., Teta A. (2014). *Quantum Dynamics of a Particle in a Tracking Chamber.* Heidelberg: Springer.

Gamow G. (1928). “Zur Quantentheorie des Atomkernes”. *Zeitschrift für Physik*, 51, pp. 204-212.

Heisenberg W. (1930). *The Physical Principles of the Quantum Theory*. Chicago: The University of Chicago Press.

Heisenberg W. (1983). *Encounters with Einstein and Other Essays on People, Places, and Particles*. Princeton: Princeton University Press.

Mott N. F. (1929). “The wave Mechanics of α-Ray Tracks”. *Proceedings of the Royal Society of London*, A 126, pp. 79-84.

Leone M., Robotti N. (2004). “A note on the Wilson cloud chamber (1912)”. *European Journal of Physics*, 25, pp. 781-791.

Pippard B. (1998). “Sir Nevill Francis Mott, C.H. 30 September 1905-8 August 1996”. *Biographical Memoirs of the Fellows of the Royal Society*, 44, pp. 315-328.

Von Neumann J. (1932). *Mathematische Grundlagen der Quantenmechanik*. Berlin: Springer.

Wheeler J.A., Zurek W.H. (eds.) (1983). *Quantum Theory and Measurement*. Princeton: Princeton Legacy Library.